University: Hong Kong Baptist University

Unit of Assessment (UoA): 11 Mathematics & Statistics

Title of case study: Mathematical Software for Constructing Scrambled Digital Sequences

(1)   Summary of the impact

Constructing scrambled digital sequences underpins an important component in randomized quasi-Monte Carlo methods and it can be applied to many financial applications starting from 2003. Our research has provided the state of the art algorithms for constructing scrambled digital sequences. The algorithms have been implemented commercially in MATLAB and the Numerical Algorithms Group (NAG) Ltd Library after 2003; as well as in open source packages such as Python. Both MATLAB and NAG have included the algorithms during the assessment period (2013-2019). Koi Investment Partners International starting from January 2017, has enjoyed direct impact through improved products and increased revenues in the evaluation of trading performance of energy-relevant financial products. For specific assets, it is found that the efficiency of the evaluation were enhanced by double digit percentages, and profitability of the trading for periods were increased similarly. On the other hand, the indirect impact is for the users of the algorithms in MATLAB and NAG, leading to acceleration in the pace of application worldwide.

(2)   Underpinning research

The impact is based on research that took place in the unit of assessment with the first major publication in 2003. The key researchers were

Fred J. Hickernell (Worked at Department of Mathematics, Hong Kong Baptist University from 1985-2005, Working at Illinois Institute of Technology from 2005-date)

Hee Sun Hong (PhD [2002] graduated at Department of Mathematics, Hong Kong Baptist University and PhD Thesis: Digital Nets and Sequences for Quasi-Monte Carlo Methods)

The algorithm on which this case study is based is to describe an efficient and effective implementation of random scrambling of deterministic (t, m, s)-nets and (t, s)-sequences by eliminating their inherent bias while retaining their low-discrepancy properties. The algorithm is implemented in code published in [P1]. This refereed journal is a vehicle for publishing efficient, tested mathematical software. The published code is written carefully to minimize the execution time by manipulating all digits of each point efficiently.

(3)   References to the research

The research has been published in international journals and has been cited. Citations are shown for the Web of Science (WOS), the Scopus (SC) and Google Scholar (GS) as of 11 June 2019.

[P1] Hong, HS; Hickernell, FJ, Algorithm 823: Implementing scrambled digital sequences ACM Transactions on Mathematical Software, 29:95-109, 2003.

https://doi.org/10.1145/779359.779360
[WOS: 61, SC: 71, GS: 136]

[P2] R. X. Yue and F. J. Hickernell, The discrepancy and gain coefficients of scrambled digital nets. Journal of Complexity, 18:135-151, 2002.
https://doi.org/10.1006/jcom.2001.0630
[WOS: 15, SC: 16, GS: 23]

[P3] H. S. Hong, F. J. Hickernell and G. Wei, The distribution of the discrepancy of scrambled digital (t, m, s)-nets, Mathematics and Computers in Simulation, 62:335-345, 2003.
https://doi.org/10.1016/S0378-4754(02)00238-0
[WOS: 3, SC: 1, GS: 7]

(4)   Details of the impact

Randomized quasi-Monte Carlo methods using scrambled (t, s)-sequences were proposed as a hybrid of Monte Carlo and quasi-Monte Carlo methods. Quasi-Monte Carlo methods, as originally conceived, are a deterministic form of highly stratified sampling that achieve essentially an $O(n^{-1})$ convergence rate to the true answer. This is significantly faster than the $O(n^{-1/2})$ convergence rate of Monte Carlo methods, even with standard variance reduction techniques. However, Monte Carlo methods have the advantage of yielding unbiased estimates. Randomized quasi-Monte Carlo methods based on scrambled (t, s)-sequences, such as Sobol' sequences provide the $O(n^{-1})$ convergence rate in an unbiased estimator. Under mild additional smoothness, scrambled (t, s)-sequences may even yield an estimator with a $O(n^{-3/2})$ convergence rate, which the deterministic (t, s)-sequences cannot achieve.

Another benefit of randomized quasi-Monte Carlo methods is that they allow error estimates. Unlike deterministic quasi-Monte Carlo methods one can estimate the error of the randomized quasi-Monte Carlo methods via the variance of replications. The original proposal for scrambled (t, s)-sequences required heavy computational cost. Our research, later called linear scrambling minimized that cost.

MATLAB, from The MathWorks (head office in Natick, Massachusetts, USA, with other offices around the world) is a commercial mathematical software system with 3 million users of MATLAB worldwide. Its associated toolboxes and Simulink, an environment for multidomain simulation, run within MATLAB and provide specialist capabilities that are widely used in engineering and finance sectors, see [S1]. The Numerical Algorithms Group (NAG) Ltd Library is a product of NAG Ltd. (head office in Oxford, UK), which has as its core business the production and sale of mathematical and statistical software companies, including libraries, see [S2].

Koi Investment Partners International (Koi) is an independent quantitative asset management company based in Hong Kong. Starting from January 2017, the company used randomized quasi-Monte Carlo methods and the MATLAB functions [S1] to the company for the evaluation of trading performance of energy-relevant financial products. For specific assets, it is found that the efficiency of the evaluation were enhanced by double digit percentages (at least 10%), and profitability of the trading starting from January 2017, were increased (at least 10%) similarly.

Pathways to Impact

F. Hickernell has a keen interest in the development of software, and has been a driving force in bringing together disparate groups with a common interest in the development of software for high dimensional problems. Hickernell's contributions to high dimensional problems were recognised definitively in 2016 by the award of the award of the Joseph Traub Prize for Achievement in Information Based Complexity. He has been very active in building bridges between the area of statistics and numerical analysis – fields that have a common interest in randomised methods, yet can seem worlds apart professionally. Because of his well-known algorithm published in [P1], the implementation of constructing scrambled digital sequences has been included in MATLAB's Fred: Statistics Toolbox [S1] and NAG [S2]. In [S1], we list MATLAB function used in class Sobolet. In [S2], we list NAG library routine document of G05YNF which states that the implementation of scrambled quasi-random sequences is based on [P1]. The related analysis of the scrambled quasi-random sequences can be found in [P2, P3].

Indeed, the majority of end users of constructing scrambled digital sequences is using the algorithms that are the focus of this case study, albeit unknowingly. We have found the non-academic impact of open source implementations of our algorithms impossible to quantify, and so instead quantify the impacts of implementations made by a local finance company: Koi Investment Partners International.

Koi Investment Partners International (Koi) founded in 2010, has an Asset Management (Type 9) license from the Hong Kong Securities and Futures Commission (SFC). Koi makes use of systematic trading strategies which eliminate any bias when decisions are made by human, either as single individual or as a team. Its strategies are well back-tested, undergo simulated trading before capital are allocated for live trading. At January 2017, the UoA has collaborated with Koi to obtain a Hong Kong Government Innovation and Technology Funding project: Deep Learning-Based Trading System for Energy-Relevant Financial Products. The project coordinator (M. Ng) introduced the randomized quasi-Monte Carlo methods and the MATLAB function [S1] to the company for the evaluation of trading performance of energy-relevant financial products. Both the efficiency of the evaluation and the profitability of the trading were increased by double digit percentages (at least 10%).

(5)   Sources to corroborate the impact

[S1] MATLAB function sobolset.m
https://www.mathworks.com/help/stats/sobolset.html

[S2] NAG library routine document of G05YNF
https://www.nag.co.uk/numeric/fl/nagdoc_fl25/html/g05/g05ynf.html
https://www.nag.co.uk/numeric/fl/nagdoc_fl25/pdf/g05/g05ynf.pdf