

**Research Assessment Exercise 2020**  
**Impact Case Study**

**University:** City University of Hong Kong |  
**Unit of Assessment (UoA):** 11 Mathematics and Statistics |

**Title of case study:** Yang-Zhang optimal volatility estimator |

**(1) Summary of the impact**

Volatility of a stock price is the most critical parameter for pricing and hedging options among professional traders. Since 1960, professional traders were looking for a volatility estimator that could deal with the reality in financial data. Over several decades, the existing estimators led to biased results. In 2000, Yang and Zhang solved this problem by deriving a new estimator. This new estimator has been well received among professional traders, as it has all the features they were looking for. Since then, this estimator, known as Yang-Zhang estimator among traders, has been implemented at many professional websites. |

**(2) Underpinning research**

Why has the Yang-Zhang volatility estimator been so well received in the financial industry?

The most important feature of financial instruments is their associated risk, which is usually measured by the volatility. Volatility is the most important parameter in modern finance and a critical parameter in many areas in finance, including risk assessment, risk management, portfolio selection, etc. Unfortunately, this important quantity is not directly observable in financial markets. In Black-Scholes formula, all quantities are known except the volatility.

Great efforts have been devoted to estimate the volatility. However, except for the inaccurate close-to-close estimator, all other previous estimators assume that “the average return of an investment is zero” (known as *no drift*) and/or that “no information will be released after the market is closed” (known as *no open jumps*). These assumptions are completely unrealistic. Estimators based on these assumptions gave biased results and the application of these estimators led to errors in pricing financial instruments and mismanagement of risks for decades. It was a long-standing problem in finance to develop an unbiased estimator that is not subject to these unrealistic assumptions. Furthermore, it is well known that estimating the drift and the measure associated with open jumps is more difficult than estimating the volatility.

For a long time, researchers and practitioners were seeking for an unbiased estimator with the following properties:

1. it allows non-zero drifts (to be consistent with reality);
2. it allows non-zero open jumps (to be consistent with reality);
3. the drift itself does not occur in the formula (to eliminate the difficulty in estimating drift);
4. the measure associated with open jumps does not occur in the formula (to eliminate the difficulty in estimating such measure);
5. it has the smallest estimation error (to obtain the most efficient estimator for given data).

Yang and Zhang completely solved this problem by taking a new approach. Except the inefficient close-to-close estimator, all other estimators developed in the literature are single-period based, namely these estimators only consider the data from single period. These

estimators were applied to a sequence of single-period data for reducing statistical error. Yang and Zhang showed that it is impossible to have a single-period based estimator that is independent of drift and open jumps. Their new approach developed a multi-period based estimator that satisfies all five properties listed above. Their estimator allows non-zero values for the drift and open jumps, but the parameters for drift and statistical property of open jumps do not appear in the estimator. In other words, the estimator is independent of drift and statistical properties of open jumps. Their new estimator is optimal with the smallest statistical error among all possible estimators with these properties. |

### **(3) References to the research**

|Dennis Yang and Qiang Zhang, "Drift-independent Volatility Estimation Based on High, Low, Open and Close Prices", *J. of Business*, 73 (2000), 477–491.

Comment: Since Yang and Zhang completely solved this long-standing problem of deriving an optimal estimator with all desirable properties that practitioners were looking for, there is no further work on this topic. |

### **(4) Details of the impact**

|The practical usefulness of Yang-Zhang estimator can be observed in the following aspects:

#### **(a) Quotations from the websites for practitioners:**

- On a webpage of stockviz.biz, it states: “Yang and Zhang were the first to derive an historical volatility estimator that has a minimum estimation error, is independent of the drift, and independent of opening gaps. This estimator is maximally 14 times more efficient than the close-to-close estimator.” [1]
- At a webpage of seekingalpha.com, it says “One of the principal advantages of the Yang-Zhang estimate is that it incorporates opening, high, and low prices for each period in addition to closing prices, which means you’ll see a higher reading on days when there are significant intraday swings, even if the closing price isn’t far from the day before. I’m inclined to regard the Yang-Zhang estimate as more reliable here” [2]
- On February 13, 2012, Colin Bennett (Head of Derivatives Strategy) and Miguel A. Gil (Equity Derivatives Strategy) of Sandander wrote: “In 2000 Yang-Zhang created the most powerful volatility measure that handles both opening jumps and drift.” “Yang-Zhang (OHLC): The most powerful volatility estimator which has minimum estimation error.” “It is up to a maximum of 14 times as efficient (for 2 days of data) as the close to close estimate.” [3]
- At a webpage of investorplace.com, Adam Warner wrote, “The most popular method for calculating historical volatility is to take the standard deviation over a certain period of the natural log of close-to-close prices. But a calculation that only considers closing prices ignores the information provided by intraday opening, high and low prices. Consider comparing other models (like Parkinson and Yang-Zhang) to get a fuller sense of how volatile an asset has been.” [4]

- At QuantConnect Coporatoon website, Laurent Keller wrote: “The (Yang-Zhang) estimator is very effective to estimate the volatility based on bar data as it takes into account the close to open gap. The estimator has the advantage of being ‘unbiased in the continuous limit, (b) independent of the drift, (c) consistent in dealing with opening price jumps. Furthermore, it has the smallest variance among all estimators with similar properties. The improvement of accuracy over the classical close-to-close estimator is dramatic for real-life time series’ (YZ, 2000). Some authors have show that, in the case of inverse volatility weighting of the position, using the YZ estimator for position sizing results in bigger profits because the turnover is reduced. If you compare the YZ volatility to the standard volatility estimator you will see that the YZ has less variation.” [5]
- On webpage of marketchameleon.com, it states: “There are many formulas to calculate volatility using OHLC, but the version that we use at MarketChameleon.com is based on the Yang-Zhang estimator. The benefit of OHLC volatility calculation is that it incorporates more elements to get a better sense of what happened throughout the day.” [6]
- At a webpage of targetedwealthsolutions.com, it states: “A way to think about realized volatility is the range of price action that happens in the market. Wild swings create higher volatility and muted price movements create lower volatility over a certain time period. High volatility is often associated with downturns in the market. We use the Yang-Zhang method to estimate historical volatility (see formula after our chart).” [7]
- Automated Trader Magazine (28 Q1 2013 issue) published an interview with Robert Kosowski. Here is extraction of the journal article on the interview:

**Automated Trader:** “Which volatility estimators did you evaluate for portfolio rebalancing of instruments, and which of these did you regard as the most effective?”

**Robert Kosowski:** “We examined a total of eight, ranging from simple standard deviation to more sophisticated estimators that took account of factors such as overnight gaps and underlying drift in the price process. Our objective was to find an estimator that introduced the minimum noise and thereby generated the most persistent volatility and in turn minimised unnecessary portfolio turnover.

‘The Yang Zhang estimator, which we finally selected as optimal for our purposes, uses a linear combination of techniques ...’ [8]

### **(b) Practitioners implemented Yang-Zhang estimator in many programming languages**

The Yang-Zhang estimator has been implemented in many different programming languages by the practitioners, including Matlab, R, Python, Excel, Java, Haskell, Pine Script Language, AmiBroker Formula Language, MetaQuotes Languange 5. See [9] for some examples.

**(c) Yang-Zhang estimator has been taught at many institutions for training practitioners** (see [10]).

**(d) Yang-Zhang volatility estimator is covered in many books for practitioners** (see [10]). |

**(5) Sources to corroborate the impact**

- [1] Yang-Zhang volatility  
<https://stockviz.biz/yang-zhang-volatility/>
- [2] Advantage of the Yang-Zhang volatility estimator  
<https://seekingalpha.com/article/234962-IMPLIED-VOLATILITY-MAY-BE-HIGHER-THAN-YOU-THINK>
- [3] Powerfulness of the Yang-Zhang volatility estimator  
[http://www.todaysgroep.nl/media/236846/measuring\\_historic\\_volatility.pdf](http://www.todaysgroep.nl/media/236846/measuring_historic_volatility.pdf)
- [4] Usefulness of the Yang-Zhang estimator in volatile market  
<https://investorplace.com/2010/06/volatility-trading-tips/>
- [5] Effectiveness of the Yang-Zhang volatility estimator  
<https://www.quantconnect.com/forum/discussion/978/yang-zhang-volatility/p1>
- [6] marketchameleon.com adopts Yang-Zhang estimator  
<https://marketchameleon.com/Learn/Volatility>
- [7] targetedwealthsolutions.com adopts the Yang-Zhang estimator  
<https://targetedwealthsolutions.com/2017/03/more-jobs-than-you-can-shake-a-stick-at/>
- [8] Evidence of the Yang-Zhang estimator being optimal choice for practitioners  
[http://www.cityu.edu.hk/ma/research/8\\_Automated\\_Trader\\_Magazine.pdf](http://www.cityu.edu.hk/ma/research/8_Automated_Trader_Magazine.pdf)
- [9] Evidence of implementation of the Yang-Zhang estimator by practitioners  
[http://www.cityu.edu.hk/ma/research/9\\_Software\\_examples.pdf](http://www.cityu.edu.hk/ma/research/9_Software_examples.pdf)
- [10] Evidence of training practitioners with the Yang-Zhang estimator  
[http://www.cityu.edu.hk/ma/research/10\\_Training\\_practitioners.pdf](http://www.cityu.edu.hk/ma/research/10_Training_practitioners.pdf)