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# The Research Grants Council of Hong Kong
# NSFC/RGC Joint Research Scheme
# Joint Completion Report

*(Please attach a copy of the completion report submitted to the NSFC by the Mainland researcher)*

## Part A: The Project and Investigator(s)

### 1. Project Title

Sparse Optimization: Algorithms and Theories

### 2. Investigator(s) and Academic Department/Units Involved

| | Hong Kong Team | Mainland Team |
|---|---|---|
| Name of Principal Investigator *(with title)* | Prof. Xiaojun Chen | Prof. Ya-xiang Yuan |
| Post | Head | Academician |
| Unit / Department / Institution | Department of Applied Mathematics<br>The Hong Kong Polytechnic University | Academy of Mathematics and Systems Science<br>Chinese Academy of Sciences |
| Co-investigator(s) *(with title)* | Prof. Xiaoming Yuan | Dr. Xin Liu |

### 3. Project Duration

| | Original | Revised | Date of RGC/ Institution Approval *( must be quoted)* |
|---|---|---|---|
| Project Start date | 1 January 2015 | | |
| Project Completion date | 31 December 2018 | | |
| Duration *(in month)* | 48 | | |
| Deadline for submission of Joint Completion Report | 31 December 2019 | | |

## Part B: The Completion Report

## 5. Project Objectives

### 5.1 Objectives as per original application

1. Develop efficient splitting algorithms for non-smooth convex sparse optimization models in vector and matrix spaces.
2. Develop efficient numerical algorithms for non-smooth non-convex (also might be non-Lipschitz) sparse optimization models in vector and matrix spaces.
3. Investigate the global convergence and worst-case iteration complexity results for the new algorithms to be proposed.
4. Apply the new algorithms to solve some applications arising in various areas such as information theory, image processing, statistical learning, etc.

5.2 Revised Objectives

Date of approval from the RGC: NA

Reasons for the change: NA

## 6. Research Outcome

Major findings and research outcome
*(maximum 1 page; please make reference to Part C where necessary)*

The PIs and Co-PIs have developed efficient algorithms for non-smooth convex sparse matrix optimization problems. Such problems have many important applications in various areas such as information theory, image processing, statistical learning, etc.

We have published six papers in top/major journal. Two papers have been accepted for publication and one joint paper is under review.

In the first paper, we show that under certain conditions, the second order optimal necessary condition implies global optimality for a class of matrix low-rank factorization models.

In the second paper, we propose a new first-order framework for orthogonal constrained optimization problems. Based on this new framework, we develop two types of algorithms: gradient reduction based algorithms and column-wise block coordinate descent algorithms. Preliminary experiments illustrate that our new framework is of great potential.

In the third paper, we established some new properties for the Łojasiewicz exponent of the quadratic sphere constrained optimization problem.

In the fourth paper, we present new complexity results of partially-separable convexly-constrained optimization with non-Lipschitzian singularities.

In the fifth paper, we proposed an exact penalty method for semidefinite-box constrained low-rank matrix optimization problems.

In the sixth paper, we developed a new continuous optimization model for spectral Clustering.

The other three papers focus on ADMM algorithm. We studied the convergence of the direct extension of ADMM for three-block separable convex minimization models with one strongly convex function, a class of ADMM-based algorithms for three-block separable convex programming, and convergence of the symmetric version of ADMM with larger step sizes.

Potential for further development of the research and the proposed course of action *(maximum half a page)*

Our research outcome will enhance the applications of sparse optimization in engineering and science.

## 7. The Layman's Summary

*(describe in layman's language the nature, significance and value of the research project, in no more than 200 words)*

Many interesting models arising in a variety of application domains such as information theory, image processing, statistical learning, medical imaging, bioinformatics, electronic commerce, computer vision, share the feature of seeking a solution with the sparsity structure of an optimization model in a vector, matrix or tensor space. Consequentially, a number of sparse optimization models arise widely in the literature. Mathematically, these models may have the difficulty of non-smoothness, non-Lipschitzness, and even non-convexity. Moreover, in the era with big data where the dimensionality of models increases extremely rapidly, these sparse optimization models usually have another common difficulty of the high dimensionality. All these difficulties critically urge optimizers to work intensively on the algorithmic aspects of these sparse optimization models and thus develop efficient numerical algorithms with affirmative theoretical supports; this is the main purpose of this project. We systematically investigated how to develop efficient numerical algorithms for some sparse optimization models with strong application backgrounds; and analyzed the corresponding theoretical properties such as the convergence and convergence rates of these algorithms. The algorithms can also be applied to solve some real-life sparsity-driven applications. The outcome of this project can enhance the interaction between optimization and other disciplines as mentioned; and provide faster algorithms for practitioners in the mentioned areas.

## Part C: Research Output

### 8. Peer-reviewed journal publication(s) arising directly from this research project

*(Please attach a copy of each publication and/or the letter of acceptance if not yet submitted in the previous progress report(s). All listed publications must acknowledge RGC's funding support by quoting the specific grant reference.)*

| The Latest Status of Publications | | | | Author(s) | Title and Journal/ Book | Submitted to RGC | Attached to this report | Acknowledged the support of this Joint Research Scheme | Accessible from the institutional repository |
|---|---|---|---|---|---|---|---|---|---|
| Year of publication | Year of Acceptance *(For paper accepted but not yet published)* | Under Review | Under Preparation *(optional)* | ***(bold** the authors belonging to the project teams and denote the corresponding author with an asterisk*)* | *(with the volume, pages and other necessary publishing details specified)* | *(indicate the year ending of the relevant progress report)* | *(Yes or No)* | *(Yes or No)* | *(Yes or No)* |
| 2017 | | | | H. Wang **X. Liu, X. Chen Y. Yuan** | SNIG Property of Matrix Low-rank Factorization Model/ J. Computational Mathematics, 36(2018), 374-390. | yes | yes | yes | yes |
| 2018 | | | | B. Gao **X. Liu, X. Chen Y. Yuan** | A New First-order Algorithmic Framework for Optimization Problems with Orthogonality Constraints / SIAM J. Optimization, 28(2018) 302-332 . | yes | yes | yes | yes |
| | | 2018 | | B. Gao **X. Liu, X. Chen Y. Yuan** | On the Łojasiewicz Exponent of the Quadratic Sphere Constrained Optimization Problem/ Mathematics of Operations Research (under review) | yes | yes | yes | yes |
| | 2018 | | | **X. Chen** P. Toint H. Wang | Complexity of partially-separable convexly-constrained optimization with non-Lipschitzian singularities /SIAM J. Optimization | yes | yes | yes | yes |
| | 2018 | | | T. Liu Z. Lu **X. Chen** Y. Dai | An exact penalty method for semidefinite-box constrained low-rank matrix optimization problems/IMA J. Numer. Anal. | yes | yes | yes | yes |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 2018 | | | | **X. Liu** M. Ng R.Zhang Z. Zhang | A new continuous optimization model for spectral clustering, /Math. Numer.Sinica, 40(2018), 354-366. | yes | yes | yes | yes |
| 2017 | | | | X. Cai D. Ha**n** **X. Yuan** | On the convergence of the direct extension of ADMM for three-block separable convex minimization models with one strongly convex function/Comput. Optim. Appl. 66(2017), 39-73 | yes | yes | yes | yes |
| 2018 | | | | B. He **X. Yuan** | A class of ADMM-based algorithms for three-block separable convex programming/ Comput. Optim. Appl. 70(2018), 791-826. | yes | yes | yes | yes |
| 2016 | | | | B. He F. Ma **X. Yuan** | Convergence study on the symmetric version of ADMM with larger step sizes/SIAM J. Imaging Sci. 9(2016),1476-1501. | yes | yes | yes | yes |

**9. Recognized international conference(s) in which paper(s) related to this research project was/were delivered** *(Please attach a copy of each delivered paper. All listed papers must acknowledge RGC's funding support by quoting the specific grant reference.)*

| Month/Year/ Place | Title | Conference Name | Submitted to RGC *(indicate the year ending of the relevant progress report)* | Attached to this report *(Yes or No)* | Acknowledged the support of this Joint Research Scheme *(Yes or No)* | Accessible from the institutional repository *(Yes or No)* |
|---|---|---|---|---|---|---|
| 12-15 July 2015 / Pittsburgh, USA | SNIG Property of Matrix Low-rank Factorization Model | The 22nd International Symposium on Mathematical Programming | 2016 | No | Yes | Yes |
| 6-11 August 2016 / Tokyo Japan | A New First-order Framework for Orthogonal Constrained Optimization Problems | The Fifth International Conference on Continuous Optimization | 2016 | No | Yes | Yes |

| | | | | | | |
|---|---|---|---|---|---|---|
| 1-6 July 2018 / Bordeaux, France | On the Łojasiewicz Exponent of Quadratic Minimization with Sphere Constraint | The 23rd International Symposium on Mathematical Programming | No | Yes | Yes | Yes |

## 10. Student(s) trained *(Please attach a copy of the title page of the thesis.)*

| Name | Degree registered for | Date of registration | Date of thesis submission/ graduation |
|---|---|---|---|
| Wang Hong | PhD | August 2013 | December 2016 |

## 11. Other impact *(e.g. award of patents or prizes, collaboration with other research institutions, technology transfer, etc.)*

The two teams have visited each other several times every year during 2015-2018. The purposes of visits were to conduct joint research on this project, and organized joint conferences/workshops. In the last 13 years, the two teams have conducted excellent collaboration. In 2017, the PI of the Hong Kong Team and that of the Mainland Team applied for the Chinese Academy of Sciences (CAS), Academy of Mathematics and Systems Science (AMSS)-PolyU Joint Laboratory of Applied Mathematics. In October 2018, CAS has approved the CAS AMSS-PolyU Joint Laboratory of Applied Mathematics. The PIs of this project are the directors of the laboratory. This joint laboratory will enhance the collaboration between applied mathematicians in Hong Kong and the Mainland.