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The Research Grants Council of Hong Kong
NSFC/RGC Joint Research Scheme
Joint Completion Report

(Please attach a copy of the completion report submitted to the NSFC by the Mainland researcher)

Part A: The Project and Investigator(s)

1. Project Title

Non-Convex Optimization for Robust Sparse Recovery: Fast Algorithms and Theoretical Analysis

2. Investigator(s) and Academic Department/Units Involved

	Hong Kong Team	Mainland Team
Name of Principal Investigator *(with title)*	Prof So Hing-cheung	Prof Gu Yuantao
Post	Professor	Professor
Unit / Department / Institution	Department of Electrical Engineering, City University of Hong Kong	Department of Electronic Engineering, Tsinghua University
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Co-investigator(s) *(with title and institution)*		

3. Project Duration

	Original	Revised	Date of RGC/ Institution Approval *(must be quoted)*
Project Start date	01-Jan-2016		
Project Completion date	31-Dec-2019		
Duration *(in month)*	48		
Deadline for Submission of Completion Report	31-Dec-2020		

<u>Part B: The Completion Report</u>

5. Project Objectives

5.1 Objectives as per original application

1. Devise fast and robust sparse recovery solutions. We will robustify two conventional approaches, which correspond to solving convex unconstrained/constrained optimization problems, via introducing possibly non-convex sparsity-inducing and noise-resistant functions in the problem formulation. In particular, the non-convex -norm function with $0 < p < 1$ will be exploited in the algorithm development.

2. Extend the algorithm development to handle complex-valued observations and produce the distributed and/or parallel implementations of the derived solvers.

3. Produce the theoretical analysis for the developed sparse recovery algorithms. Important performance measures including computational complexity, local and global convergence as well as conditions of exact recovery, will be examined.

4. Apply the devised non-convex based methodology to important problems including spectral estimation, source localization, image denoising, magnetic resonance imaging and social network analysis, and compare with the corresponding start-of-the-art techniques in terms of recovery performance, convergence, robustness and computational complexity.

5.2 Revised Objectives

Nil.

6. Research Outcome

Major findings and research outcome
(maximum 1 page; please make reference to Part C where necessary)

We have addressed the problem of robust sparse recovery by exploring the behavior of non-convex and non-smooth sparseness measure by designing and verifying two novel algorithms [J4, J3]. In [J4], we consider that the optimization cost function is a linear combination of a non-smooth sparsity-inducing term and an ℓ_2-norm as the metric for the residual error. To handle the nondifferentiable cost functions, locally competitive algorithm and forward Euler discretization method are exploited to approximate the non-smoothness. Alternating Direction Method of Multipliers (ADMM) is then applied as the solver of the resultant smooth optimization problem, and Nesterov acceleration is integrated for speeding up the computation process. In [J3], we turn to the robust sparse recovery by specializing the non-smooth sparseness measure as a class of weakly convex functions and replacing the metric for fidelity from ℓ_2-norm to ℓ_1-norm. A slack variable is introduced to guarantee the convexity of the equivalent optimization problem in each block of variables and an efficient algorithm is derived for minimizing the surrogate Lagrangian based on the ADMM. The use of weakly convex sparseness measure guarantees that this novel robust sparse recovery formulation attains the global optimum solution. Compared with several state-of-the-art algorithms, our new methods obtain better recovery performance particularly in the presence of impulsive noise.

We have focused on the theoretical analysis for -norm and weakly convex function as both sparseness penalty and robust fidelity measures [J2, J5, J7]. In [J2], we start investigating the sparse recovery conditions and performance bounds for -minimization. Based on our devised Null Space Constant (NSC) upper bound, which outperforms the state-of-the-art result, we provide a new Restricted Isometry Constant (RIC) upper bound dependent on the sparsity level as a sufficient condition for precise recovery, and it is tighter than the existing bound for small sparsity level. Then we study the largest choice of for the -minimization problem to recover any -sparse signal, and the largest recoverable for a fixed . In [J5], we directly analyze the performance bound of a general optimization problem for robust sparse signal recovery, including many existing works as concrete instances, by a freshly defined Double NSC (DNSC), and its solution is proved to be able to robustly reconstruct the sparse signal under mild conditions. Moreover, for computational tractability, weakly convex sparsity-inducing penalties are applied to the general problem, and properties of the solution to the resultant non-convex problem are further studied. Based on these properties, an algorithm named Robust Projected Generalized Gradient (RPGG) is devised to solve the weakly convex problem. Theoretical results prove that the sparse signal can be precisely reconstructed by RPGG from compressive measurements with sparse noise or robustly recovered from those with impulsive noise. In [J7], we combine the provable behavior of weakly convex sparseness measure and the efficient operation of deep neural networks in designing a network named Learning Proximal Operator Method (LePOM) for sparse recovery. Theoretical analysis of this network inspires us to further reduce the number of parameters and arrive at proposing the Analytical LePOM (ALePOM). ALePOM determines most parameters by solving an optimization problem and significantly reduces the number of parameters. It is proved that if the signal is sufficiently sparse, ALePOM converges linearly. Simulations confirm our analyses and demonstrate that the proposed methods outperform state-of-the-art sparse recovery algorithms and neural network-based methods. Based on deep learning, recently we have also developed a proximal gradient algorithm for fast sparse matrix recovery and its superiority over several conventional schemes is demonstrated [S1].

Furthermore, we have applied our developed methodologies in improving the solution for the problem of direction-of-arrival (DOA) estimation [J6, J1]. In [J6], we introduce weakly convex sparseness measure in robust DOA estimation via low-rank matrix approximation. Compared with several existing algorithms, the proposed methods enjoy smaller computational complexity, comparable DOA estimation performance against impulsive noise and requiring no *a priori* information of the source number. While in [J1], we develop a novel objective function regularized by the nonconvex sparsity-inducing penalty for off-grid DOA estimation and utilize alternating minimization to tackle this joint sparse representation of the signal recovery and perturbation matrix. Numerical simulations verify the effectiveness of the proposed method, which achieves more accurate DOA estimation performance and faster implementation than the conventional sparsity-aware and state-of-the-art off-grid schemes.

Potential for further development of the research and the proposed course of action *(maximum half a page)*

Recently, the roaring success of deep learning contributes to the advancement of sparse signal recovery. Many conducted works have been focusing on neural networks based on unfolding iterative algorithms. We recognize that greedy pursuit algorithms usually require abundant samples to guarantee the recovery. The state-of-the-art global optimization algorithms for sparse recovery are usually computationally inefficient. In contrast, the methods based on the neural network unfolding technique have higher computational efficiency. However, to the best of our knowledge, the research in learning-based robust sparse recovery is still rare or lacks applicability due to the tough choices of hyper-parameters. Along this direction, there are a lot of concrete works need to be done including algorithm development and theoretical analysis.

7. The Layman's Summary

(describe in layman's language the nature, significance and value of the research project, in no more than 200 words)

Sparse recovery refers to extracting a high-dimensional vector with few nonzero entries from a small number of linear measurements. It has been a core topic in science and engineering because many real-world signals have a sparse representation in some basis. As an intensive field of research, there are still obstacles need to be overcome to further enhance its practicality. One key issue is to solve large-scale problems in big data analytics where the number of variables is enormous, implying the need of computationally attractive solutions. Another challenge is to recover the sparse signals from as few observations as possible. A representative example is in magnetic resonance imaging where significant scan time reduction means benefits for patients and health care economics. Furthermore, many existing sparse recovery algorithms assume that the measurement noise is Gaussian distributed. However, the occurrence of non-Gaussian impulsive noise is common, and thus these standard solvers might be unable to provide reliable performance in such scenarios. In this research, we utilize non-convex sparsity-inducing and noise-resistant functions in devising efficient, robust, and provable algorithms to recover sparse signals in non-Gaussian noise environment with minimum observations. We expect that our research results can provide a significant value in sparse recovery.

Part C: Research Output

8. Peer-reviewed journal publication(s) arising directly from this research project

(Please attach a copy of each publication and/or the letter of acceptance if not yet submitted in the previous progress report(s). All listed publications must acknowledge RGC's funding support by quoting the specific grant reference.)

The Latest Status of Publications				Author(s) *(**bold** the authors belonging to the project teams and denote the corresponding author with an asterisk*)*	Title and Journal/ Book *(with the volume, pages and other necessary publishing details specified)*	Submitted to RGC *(indicate the year ending of the relevant progress report)*	Attached to this report *(Yes or No)*	Acknowledged the support of this Joint Research Scheme *(Yes or No)*	Accessible from the institutional repository *(Yes or No)*
Year of publication	Year of Acceptance *(For paper accepted but not yet published)*	Under Review	Under Preparation *(optional)*						
2017 **[J1]**				**Q.Liu***, **H.C.So**, **Y.Gu**	Off-grid DOA estimation with nonconvex regularization via joint sparse representation, *Signal Processing*, vol.140, pp.171-176, Nov. 2017	2017	Yes	Yes	Yes
2018 **[J2]**				**C.Yang, X. Shen,** H.Ma, **Y.Gu***, **H.C.So**	Sparse recovery conditions and performance bounds for ℓ_p-minimization, *IEEE Transactions on Signal Processing*, vol.66, pp.5014-5028, Oct. 2018		Yes	Yes	Yes

2018 **[J3]**				**Q.Liu***, **C.Yang,** **Y.Gu**, **H.C.So**	Robust sparse recovery via weakly convex optimization in impulsive noise, *Signal Processing*, vol.152, pp.84-89, Nov. 2018		Yes	Yes	Yes
2019 **[J4]**				**Q.Liu**, **Y.Gu**, **H.C.So***	Smoothed sparse recovery via locally competitive algorithm and forward Euler discretization method, *Signal Processing*, vol.157, pp.97-102, Apr. 2019		Yes	Yes	Yes
2019 **[J5]**				**C.Yang,** **X.Shen,** H.Ma, B.Chen, **Y.Gu**, **H.C.So***	Weakly convex regularized robust sparse recovery methods with theoretical guarantees, *IEEE Transactions on Signal Processing*, vol.67, no.13, pp.5046-5061, Oct. 2019		Yes	Yes	Yes
2019 **[J6]**				**Q.Liu**, **Y.Gu**, **H.C.So***	DOA estimation in impulsive noise via low-rank matrix approximation and weakly convex optimization, *IEEE Transactions on Aerospace and Electronic Systems*, vol.55, no.6, pp.3603-3616, Dec. 2019		Yes	Yes	Yes
[J7]	2020			**C.Yang,** **Y.Gu***, B.Chen, H.Ma, **H.C.So**	Learning proximal operators for sparse recovery with theoretical guarantee, *IEEE Transactions on Signal Processing, accepted*		Yes	Yes	No
[S1]		2020		**C.Yang**, **Y.Gu***, B.Chen, H.Ma, **H.C.So**	2D learned proximal gradient algorithm for fast sparse matrix recovery, *submitted to IEEE Transactions on Circuits and Systems II: Express Briefs*		Yes	Yes	No

9. **Recognized international conference(s) in which paper(s) related to this research project was/were delivered** *(Please attach a copy of each delivered paper. All listed papers must acknowledge RGC's funding support by quoting the specific grant reference.)*

Month/Year/ Place	Title	Conference Name	Submitted to RGC *(indicate the year ending of the relevant progress report)*	Attached to this report *(Yes or No)*	Acknowledged the support of this Joint Research Scheme *(Yes or No)*	Accessible from the institutional repository *(Yes or No)*

10. Student(s) trained *(Please attach a copy of the title page of the thesis.)*

Name	Degree registered for	Date of registration	Date of thesis submission/ graduation
Qi Liu	Ph.D.	Aug. 16	Aug. 19

11. Other impact *(e.g. award of patents or prizes, collaboration with other research institutions, technology transfer, etc.)*

During this research, we have the opportunity to collaborate with Prof. Badong Chen at Xi'an Jiaotong University. Inspired by the -norm idea, a patent on robust matrix factorization has been granted, namely, W.-J. Zeng, **H.C. So** and J. Chen, "Systems and methods for robust low-rank matrix approximation," U.S. Patent No. 10,229,092, 2019