RGC Ref.: N_CityU102/12 NSFC Ref. : 11261160485
(please insert ref. above)

The Research Grants Council of Hong Kong
NSFC/RGC Joint Research Scheme
Joint Completion Report

(Please attach a copy of the completion report submitted to the NSFC by the Mainland researcher)

Part A: The Project and Investigator(s)

1. Project Title

Mathematical Theories of Some Kinetic and Fluid Models

2. Investigator(s) and Academic Department/Units Involved

	Hong Kong Team	Mainland Team
Name of Principal Investigator *(with title)*	Prof Tong YANG	Prof Huijiang ZHAO
Post	Chair Professor	Associate Dean
Unit / Department / Institution	Department of Mathematics/ Citv Universitv of Hong Kong	School of Mathematics and Statistics/ Wuhan University
Contact Information	*matyang@cityu.edu.hk*	
Co-investigator(s) *(with title and institution)*		

3. Project Duration

	Original	Revised	Date of RGC/ Institution Approval *(must be quoted*)
Project Start date	01-Jan-2013		
Project Completion date	31-Dec-2016		
Duration *(in month)*	48		
Deadline for Submission of Completion Report	31-Dec-2017		

Part B: The Completion Report

5. Project Objectives

5.1 Objectives as per original application

1. Well-posedness theories for some complex kinetic systems;
2. Exterior problem for kinetic equations without angular cutoff;
3. Formation of patterns and propagation of singularities;
4. Global solutions to fluid models with large data.

5.2 Revised Objectives

Date of approval from the RGC: _____

Reasons for the change: _____

_____

6. Research Outcome

Major findings and research outcome
(maximum 1 page; please make reference to Part C where necessary)

The major research findings and outcome were written in seven research papers, six of them have been published in refereed international journals, such as Journal of European Mathematical Society, Archive for Rational Mechanics and Analysis. Some detail of these papers is given as follows.

The first paper is concerned with the Cauchy problem of the one-dimensional compressible Navier-Stokes equations with degenerate temperature dependent transport coefficients that satisfy conditions from the consideration in kinetic theory. A result on the existence and uniqueness of globally smooth non-vacuum solution is obtained and the perturbation can be large if the adiabatic constant is close to one.

The second paper aims to estimate the thickness of the boundary layer for the planar MHD system with vanishing shear viscosity. Under some conditions on the initial and boundary data, we show that the thickness is of the order close to the square-root of the shear viscosity coefficient.

In the third paper, both the global well-posedness for large data and the vanishing shear viscosity limit with boundary layer to the compressible Navier-Stokes system with cylindrical symmetry are studied under a general condition on the heat conductivity coefficient. The thickness of the boundary layer is proved to be almost optimal. Moreover, the optimal L1 convergence rate in term of shear viscosity is obtained for the angular and axial velocity components.

The spectrum structures and behaviors of the Vlasov-Maxwell-Boltzmann systems for both two species and one species are studied in the fourth paper. The analysis shows the effect of the Lorentz force induced by the electro-magnetic field leads to some different structure of the spectrum from the classical Boltzmann equation and the closely related Vlasov-Poisson-Boltzmann system. And the major difference between the two-species VMB model and one-species VMB model are given. The structure in high frequency illustrates the hyperbolic structure of the Maxwell equation. Furthermore, the long time behaviors and the optimal convergence rates to the equilibrium are established.

Note that the Boltzmann H-theorem implies that the solution to the Boltzmann equation tends to the equilibrium, that is, a Maxwellian when time tends to infinity. This has been proved in varies settings when the initial energy is finite. However, when the initial energy is infinite, the time asymptotic state is no longer described by a Maxwellian, but a self-similar solution obtained by Bobylev-Cercignani. The purpose of the fifth paper is to rigorously justify this for the spatially homogeneous problem with Maxwellian molecule type cross section without angular cutoff.

The spectrum structures for the Vlasov-Poisson-Fokker-Planck and Vlasov-Poisson-Landau systems are studied in the sixth paper together with the optimal decay estimates on the convergence in time to a global equilibrium state in the small perturbation framework.

The last paper is about the Cauchy problem on the one dimensional inhomogeneous non-cutoff Kac equation. Based on the analysis on the linearized operator obtained by Lerner et al, we first prove the existence of global solution to the equation around a global Maxwellian by combining two sets of macro-micro decomposition. Then by using the dissipative norm of the linearized operator in the fractional Hermite-Sobolev space and by using the perturbation theory, the spectrum structure of the linearized Kac equation will be given. Based on this, the optimal time decay estimate for the nonlinear Kac equation is obtained.

Potential for further development of the research and the proposed course of action *(maximum half a page)*

The objectives of this project are to investigate some fluid and kinetic equations about their well-posedness and large time behavior. With the results obtained in this project, studies can be further pursed in the following directions. First of all, even though the well-posedness theories for fluid models with large initial data have been intensive studied, many challenging problems remain unsolved. One of the most outstanding problems is the millennium problem on Navier-Stokes equations. Even though it is not clear that the research of this project is related to this famous problem, the analysis and approach developed can be applied to some related problems with large perturbation. Secondly, the spectrum structures of the linearized equations around a global equilibrium state have been studied for several kinetic systems in the project With this, one can further study the Green functions and the pointwise structure of solutions to these kinetic systems. In terms of large time behavior of the Boltzmann equation with infinite energy that does not follow from the famous H-theorem, one can further study the cases of soft potentials. In addition, it is noted that there is no global in time self-similar solution of infinite energy for the spatially homogeneous Boltzmann equation with hard potential. On the other hand, the solution is expected to gain moments immediately in positive time. Hence, how to establish the well-posedness of the measure value solution with infinite energy initial data is an interesting

problem and it may reveal some new mechanism about the sudden change of solution behavior of kinetic equations.

7. The Layman's Summary

(describe in layman's language the nature, significance and value of the research project, in no more than 200 words)

Physics study of gas and fluid provides many mathematical models. For example, in the macroscopic scale, the Euler equations and Navier-Stokes equations are the most famous systems. In the microscopic scale, the motion is governed by the coupled Newton equations. Although the Newton equation is the first principle of the classical mechanics, it is not of practical use when the number of the equations is enormous. Thus, the kinetic theory that gives the mesoscopic description is a key theory that links the microscopic and macroscopic models. The most fundamental kinetic equation is the Boltzmann equation. In fact, the relation between the Boltzmann equation and the classical systems of fluid dynamics is revealed by the Hilbert and Chapman-Enskog expansions. And the study on this kind of limiting process is related to the Hilbert's sixth problem, "Mathematical treatment of the axioms of physics".

This project aims to study the solution behavior described by either fluid system or kinetic equations so that the problems to be investigated have strong physical background and challenging mathematical difficulties. The results obtained not only enrich the mathematical theories, but also shed some light on the explanation of some physical phenomena.

Part C: Research Output

8. Peer-reviewed journal publication(s) arising directly from this research project

(Please attach a copy of each publication and/or the letter of acceptance if not yet submitted in the previous progress report(s). All listed publications must acknowledge RGC's funding support by quoting the specific grant reference.)

The Latest Status of Publications				Author(s) ***(bold** the authors belonging to the project teams and denote the corresponding author with an asterisk*)*	Title and Journal/ Book *(with the volume, pages and other necessary publishing details specified)*	Submitted to RGC *(indicate the year ending of the relevant progress report)*	Attached to this report *(Yes or No)*	Acknowledged the support of this Joint Research Scheme *(Yes or No)*	Accessible from the institutional repository *(Yes or No)*
Year of publication	Year of Acceptance *(For paper accepted but not yet published)*	Under Review	Under Preparation *(optional)*						

2014				Hong Xia Liu, **Tong Yang**, Huijiang Zhao and Qingyang Zou	One-dimensional compressible Navier-Stokes equations with temperature dependent transport coefficients and large data, SIAM J. Math. Anal. Vol 46, No. 3, 2185-2228		Yes	Yes	Yes
2015				Xulong Qin, **Tong Yang**, Zheng-an Yao and Wenshu Zhou	A study on the boundary layer for planar magnetohydrodynamics system, Acta Mathematica Scientia, 35B(4), 787-806		Yes	Yes	Yes
2015				Xulong Qin, **Tong Yang**, Zheng-an Yao and Wenshu Zhou	Vanishing shear viscosity and boundary layer for the Navier-Stokes equations with cylindrical symmetry, Arch. Rational Mech. Anal. 216, 1049-1086		Yes	Yes	Yes
2016				Hai-Liang Li, **Tong Yang** and Mingying Zhong	Spectrum structure and behaviors of the Vlasov-Maxwell-Boltzmann systems, SIAM J. Math. Anal. Vol. 48, no. 1, 595-669		Yes	Yes	Yes

2017				Yoshinori Morimoto, **Tong Yang** and Huijiang Zhao	Convergence to self-similar solutions for the homogeneous Boltzmann equation, J. Eur. Math. Soc. 19, 2041-2067		Yes	Yes	Yes
2016				H-L Li, J-W Sun, **T Yang** and M-Y Zhong	Large time behavior of solutions to Vlasov-Poisson-Landau (Fokker-Planck) equations, Sci. Sin. Math, 46, 981-1004		Yes	Yes	Yes
		v		**Tong Yang** and Hongjun Yu	Global solution for the spatially inhomogeneous non-cutoff Kac equation		Yes	Yes	Yes

9. Recognized international conference(s) in which paper(s) related to this research project was/were delivered

(Please attach a copy of each delivered paper. All listed papers must acknowledge RGC's funding support by quoting the specific grant reference.)

Month/Year/ Place	Title	Conference Name	Submitted to RGC *(indicate the year ending of the relevant progress report)*	Attached to this report *(Yes or No)*	Acknowledged the support of this Joint Research Scheme *(Yes or No)*	Accessible from the institutional repository *(Yes or No)*
Aug/2015	Measure valued solutions to the Boltzmann equation	The 8th international congress on industrial and applied mathematics		Yes	Yes	Yes

10. Student(s) trained

(Please attach a copy of the title page of the thesis.)

Name	Degree registered for	Date of registration	Date of thesis submission/ graduation

11. Other impact *(e.g. award of patents or prizes, collaboration with other research institutions, technology transfer, etc.)*

NA