PROCORE - FRANCE/HONG KONG JOINT RESEARCH SCHEME COMPLETION REPORT

Project Reference Number

F-HK39/11T

Project Title

Vector valued Nevanlinna theory and systems of algebraic differential equations

Particulars

	Hong Kong team	French team
Name of Project Co-ordinator (with title)	English: Dr. Tuen Wai Ng Chinese: 吳端偉博士	Prof. Robert Conte
Name of Co-Investigator (if any)	English: Chinese:	
Institution or Institutional affiliation	[ ] CityU [ ] CUHK [ ] HKBU [ ] HKIEd [X] HKU [ ] HKUST [ ] LU [ ] PolyU	[X] CEA [ ] CNRS No. [ ] INFREMER [ ] University of Others: [ ] INRA [ ] INRIA [ ] INSERM No.
Other project team members (if any)	WONG Kowk Kin (MPhil student)	Anton Peshkov (PhD student)

Funding Period

	1st year	2nd year (if applicable)
Start Date	01-JAN-2012	01-JAN-2013
Completion Date	31-DEC-2012	31-DEC-2013

Objective(s) as per original application

1. To develop a vector valued version of Clunie Lemma for vector valued Nevanlinna theory in complex function theory.
2. To study the zero distribution and growth rate of the solutions of certain systems of algebraic differential equations by using the vector valued version of Clunie Lemma.
3. To develop some algebraic method to find exact meromorphic solutions of some coupled differential equations related to some important nonlinear partial differential equations.

[Please attach relevant document(s)]

i) Outline of proposed research and results obtained

To prove that any meromorphic solution of certain differential equation is necessarily elliptic or degenerate elliptic following an approach first developed by Eremenko. To apply this method, one needs to show that two required conditions must be satisfied. We established a systematic way to check these two conditions.

The work we have carried out was to show that a very important equation in physics and fluid mechanics, called complex quintic Ginzburg-Landau equations actually satisfies the two required conditions of Eremenko's method. Then we were able to a new solution which is elliptical non-degenerate [1,2]. The same method was also applied to obtain new exact meromorphic solutions of the real cubic Swift-Hohenberg equation. These results are published in a joint paper [3] with Mr. K.K. Wong who is a MPhil student of the Hong Kong PI.

The two PIs also supervised a HKU PhD student Mr. Chengfa Wu who will complete his PhD degree this summer. Three of us are able to apply Eremenko's method to study systems of ODEs coming from the explicit construction of Bryant Solitons in the theory of Ricci flow. In fact, we were able to find new and explicit Bryant Solitons [4].

ii) Significance of research results

The new elliptic solution we found for complex quintic Ginzburg-Landau equation and the real cubic Swift-Hohenberg equation should be interested to physicists and engineers, while the new and explicit Bryant Solitons should be interested to mathematicians working in differential geometry.

iii) Research output

1. R.~Conte and T.-W.~Ng,
Detection and construction of an elliptic solution
to the complex cubic-quintic Ginzburg-Landau equation,
Teoreticheskaya i Matematicheskaya Fizika {\bf 172} (2012) 224--235.
Theor.~Math.~Phys.~{\bf 172} (2012) 1073--1084.
http://arxiv.org/abs/1204.3028
(presented at a conference in Lecce, Italy, 13-14 September 2011)
http://www.le.infn.it/~beccaria/Boiti/Welcome.html

2. R.~Conte and T.W.~Ng,
Meromorphic traveling wave solutions of
the complex cubic-quintic Ginzburg-Landau equation,
Acta Applicandae Mathematicae {\bf 122} 153--166 (2012).
http://arxiv.org/abs/1204.3032
(presented at WASCOM, 12-18 June 2011, Brindisi)

3. R.~Conte, T.-W.~Ng and Kwok-Kin Wong,
Exact meromorphic solutions of the real cubic Swift-Hohenberg equation,
Studies in Applied Mathematics {\bf 129} (2012) 117--131.
http://arXiv.org/abs/1202.3579

4. R.~Conte, T.-W.~Ng and C.F Wu,
New explicit Bryant solitons, (2014), preprint.

The French PI gave several seminars on our work during his visits to HKU, City U, HKUST. The Hong Kong PI gave a talk on the results of new Bryant Solitons at Xiamen University in December, 2013.

iv) Potential for or impact on further research collaboration

It is expected that our method finding new explicit Bryant solitons will also allow us to give new and explicit constructions of various concept in differential geometry. For example, in the near future, we will apply our method to study the Yamabe solitons.