RGC Ref. No.:
UGC/FDS14/P03/14
(please insert ref. above)

RESEARCH GRANTS COUNCIL
COMPETITIVE RESEARCH FUNDING SCHEMES FOR
THE LOCAL SELF-FINANCING DEGREE SECTOR

FACULTY DEVELOPMENT SCHEME (FDS)

Completion Report
(for completed projects only)

Submission Deadlines:
1. *Auditor's report with unspent balance, if any: within **six** months of the approved project completion date.*
2. *Completion report: within **12** months of the approved project completion date.*

Part A: The Project and Investigator(s)

1. Project Title

Generalized Multicriteria Programs and Their Application in Portfolio Selection Problems

2. Investigator(s) And Academic Department(s) / Unit(s) Involved

Research Team	Name / Post	Unit / Department / Institution
Principal Investigator	YU Kwok-wai / Associate Professor	Department of Mathematics and Statistics / Hang Seng Management College
Co-Investigator(s)	YANG Xiao Qi / Professor	Department of Applied Mathematics / The Hong Kong Polytechnic University
Others		

3. Project Duration

	Original	Revised	Date of RGC / Institution Approval *(must be quoted)*
Project Start Date	1 January 2015		
Project Completion Date	31 December 2016	30 June 2017	12 September 2016
Duration *(in month)*	24 months	30 months	12 September 2016
Deadline for Submission of Completion Report	31 December 2017	30 June 2018	12 September 2016

Part B: The Final Report

5. Project Objectives

5.1 Objectives as per original application

1. to investigate the piecewise linear multicriteria programs and establish the Pareto optimal solutions for generalized piecewise linear multicriteria programs;
2. to develop optimization solution methods and effective algorithms for bi-criteria portfolio optimization problems under the L_∞, VaR and CVaR risk measures, respectively.

5.2 Revised objectives

Date of approval from the RGC: _____

Reasons for the change: _____

_____

_____

1.

2.

3. ....

5.3 Realisation of the objectives
(Maximum 1 page; please state how and to what extent the project objectives have been achieved; give reasons for under-achievements and outline attempts to overcome problems, if any)

The two project objectives have been addressed.

As stated in Objective 1, the project is to investigate the piecewise linear multicriteria programs. Since there exist a large (possibly infinite) number of Pareto optimal or non-dominated solutions for multicriteria programs, it is not an easy task to find effective solutions to such programs. We have reviewed the formulations for piecewise linear functions involved in a piecewise linear multicriteria program and proposed to formulate the function by modeling its epigraph related to the Pareto cone. It should be noticed that the epigraph of the function is the union of finitely many polyhedral sets. We have studied the optimality conditions and the properties of the Pareto optimal solutions for generalized piecewise linear multicriteria programs. We have also explored and proposed several subgradient methods for solving quasiconvex optimization problems. These methodologies can be applied for solving multicriteria programs involving quasiconvex functions.

In addition, we have investigated the bi-criteria portfolio optimization problems under the L_∞, VaR and CVaR risk measures, respectively (Objective 2). In particular, we proposed an alternative L_∞ downside risk measure to replace the L_∞ risk measure in a portfolio optimization problem. According to an equivalence relation between a multicriteria linear program and its weighted sum linear programs, and by a simple transformation, our

problem can be solved by considering its weighted sum piecewise linear programs. We have studied the characteristics of efficient portfolios and efficient frontier for our portfolio optimization model and provide analytic derivation of the optimal solution set.

5.4 Summary of objectives addressed to date

Objectives *(as per 5.1/5.2 above)*	**Addressed** *(please tick)*	**Percentage Achieved** *(please estimate)*
1. to investigate the piecewise linear multicriteria programs and establish the Pareto optimal solutions for generalized piecewise linear multicriteria programs	✓	100%
2. to develop optimization solution methods and effective algorithms for bi-criteria portfolio optimization problems under the L_∞, VaR and CVaR risk measures, respectively	✓	100%

6. Research Outcome

6.1 Major findings and research outcome
(Maximum 1 page; please make reference to Part C where necessary)

The project addresses the multicriteria nature of the portfolio selection problem. A portfolio optimization model under an alternative L_∞ downside risk measure has been proposed. The problem is formulated as a bicriteria piecewise linear program and can be converted into a parametric optimization problem, i.e. a weighted sum piecewise program. We have shown how to derive the efficient portfolios and the efficient frontier analytically. One of our findings is that the portfolio with the global minimum L_∞ downside risk has the optimality strategy which recovers the so-called 1/N portfolio strategy. In our analytic derivation, we characterize the feasible portfolios in the bicriteria plane. Our main results show that the set of efficient portfolios consists of some bounded polyhedra and one unbounded polyhedron, and accordingly, the efficient frontier consists of some closed line segments and one closed half-line. Hence, the optimal solution set of our parametric optimization problem can be derived analytically.

6.2 Potential for further development of the research and the proposed course of action *(Maximum half a page)*

The multicriteria nature of the portfolio selection problem has been addressed in our project. We have investigated the bicriteria piecewise linear program for portfolio selection. Our proposed problem can be extended further to a more general case in which three or more criteria are considered in the formulation of multicriteria portfolio optimization problems. Subgradient methods can be applied for solving such problems.

7. Layman's Summary

(Describe in layman's language the nature, significance and value of the research project, in no more than 200 words)

Multicriteria programming is an important topic in multiple criteria decision making. Multicriteria programs, which are usually formulated with a number of objective functions, are known to be powerful and are used extensively in various applications in the fields of financial engineering, management science, telecommunications, transportation and logistics. For each multicriteria program, there exist a large number of Pareto optimal solutions. It is generally difficult to locate nondominated sets completely and exactly. We have investigated the optimality conditions for multicriteria programs and determine the Pareto optimal sets analytically. The results and methodologies should be extendable to other fields for solving practical problems.

We have also investigated the methodologies of piecewise linear multicriteria programs with application to portfolio selection problems. It is well known that risk management is a core issue in portfolio selection problems, and thus developing an applicable methodology will provide an optimal investment strategy while directing risk management. The ideas/methodologies will bring advantages to financial industries in Hong Kong and other countries on a global scale.

Part C: Research Output

8. Peer-Reviewed Journal Publication(s) Arising Directly From This Research Project

(Please attach a copy of the publication and/or the letter of acceptance if not yet submitted in the previous progress report(s). All listed publications must acknowledge RGC's funding support by quoting the specific grant reference.)

The Latest Status of Publications				**Author(s)** *(denote the correspond-ing author with an asterisk*)*	**Title and Journal / Book** *(with the volume, pages and other necessary publishing details specified)*	**Submitted to RGC** *(indicate the year ending of the relevant progress report)*	**Attached to this Report** *(Yes or No)*	**Acknowledged the Support of RGC** *(Yes or No)*	**Accessible from the institutional repository** *(Yes or No)*
Year of Publication	**Year of Acceptance** *(For paper accepted but not yet published)*	**Under Review**	**Under Preparation** *(optional)*						
2018				Carisa Kwok Wai YU, Yaohua HU, Xiaoqi YANG* and Siu Kai CHOY	Abstract Convergence Theorem for Quasi-convex Optimization Problems with Applications, Optimization.	NA	Yes	Yes	Yes
2018				Yan ZHANG, Yaohua HU*, Carisa Kwok Wai YU, and Jinhua WANG	Cubic Convergence of Newton-Steffen sen's Method for Operators with Lipschitz Continuous Derivative, Journal of	NA	Yes	Yes	Yes

					Nonlinear and Convex Analysis, 19(3), pp. 433-460.				
2018				Yan ZHANG, Carisa Kwok Wai YU, Ji-feng BAO, and Jinhua WANG	On Quadratical Convergence of Inexact Levenberg-Marquardt Methods under Local Error Bound Condition, Journal of Nonlinear and Convex Analysis, 19(1), pp. 123-146.	NA	Yes	Yes	Yes
2017				Yaohua HU*, Xiaoqi YANG and Carisa Kwok Wai YU	Subgradient Methods for Saddle Point Problems of Quasiconvex Optimization, Pure and Applied Functional Analysis, 2(1), pp. 83-97.	2016	Yes	Yes	Yes
2016				Yaohua HU*, Carisa Kwok Wai YU, Chong LI and Xiaoqi YANG	Conditional Subgradient Method for Constrained Quasi-convex Optimization Problems, Journal of Nonlinear and Convex Analysis, 17(10), pp. 2143-2158.	2016	Yes	Yes	Yes
2016				Yaohua HU*, Carisa Kwok Wai YU, and Chong LI	Stochastic Subgradient Method for Quasi-Convex Optimization Problems, Journal of Nonlinear and Convex Analysis, 17(4), pp. 711-724.	2016	Yes	Yes	Yes
		✓		Kaiwen MENG, Carisa Kwok Wai YU* and Xiaoqi YANG	Bicriteria Portfolio Optimization via ℓ_∞ Downside Risk Measure, Annals of Operations Research		No	Yes	No

9. Recognized International Conference(s) In Which Paper(s) Related To This Research Project Was / Were Delivered

(Please attach a copy of each conference abstract)

Month / Year / Place	Title	Conference Name	Submitted to RGC *(indicate the year ending of the relevant progress report)*	Attached to this Report *(Yes or No)*	Acknowledged the Support of RGC *(Yes or No)*	Accessible from the institutional repository *(Yes or No)*
July / 2016 / Poznan, Poland	Portfolio Optimization Models under Various Risk Measures	The 28th European Conference on Operational Research	2016	Yes	Yes	Yes
December / 2015 / Changsha, China	Bicriteria Portfolio Optimization Problem under the Minimax Rule	The 6th International Conference on Optimization and Control with Applications	2015	Yes	Yes	Yes
August / 2015 / Toronto, Canada	Bi-criteria Portfolio Optimization Problems with the Use of Conditional Value-at-Risk Risk Measure	The 50th Actuarial Research Conference	2015	Yes	Yes	Yes

10. Whether Research Experience And New Knowledge Has Been Transferred / Has Contributed To Teaching And Learning

(Please elaborate)

NA

11. Student(s) Trained

(Please attach a copy of the title page of the thesis)

Name	Degree Registered for	Date of Registration	Date of Thesis Submission / Graduation
NA			

12. Other Impact

(e.g. award of patents or prizes, collaboration with other research institutions, technology transfer, teaching enhancement, etc.)

NA

13. Public Access Of Completion Report

(Please specify the information, if any, that cannot be provided for public access and give the reasons.)

Information that Cannot Be Provided for Public Access	Reasons
NA	

RESEARCH GRANTS COUNCIL
COMPETITIVE RESEARCH FUNDING SCHEMES FOR
THE LOCAL SELF-FINANCING DEGREE SECTOR

FACULTY DEVELOPMENT SCHEME (FDS)

Completion Report - Attachment
(for completed projects only)

RGC Ref. No.:	UGC/FDS14/P03/14
Principal Investigator:	YU Kwok-wai
Project Title:	Generalized Multicriteria Programs and Their Application in Portfolio Selection Problems

Statistics on Research Outputs

	Peer-reviewed Journal Publications	Conference Papers	Scholarly Books, Monographs and Chapters	Patents Awarded	Other Research Outputs (Please specify)
No. of outputs arising directly from this research project [or conference]	6	3			