RGC Ref. No.:
UGC/FDS14/P02/17
(please insert ref. above)

RESEARCH GRANTS COUNCIL
COMPETITIVE RESEARCH FUNDING SCHEMES FOR THE LOCAL SELF-FINANCING DEGREE SECTOR

FACULTY DEVELOPMENT SCHEME (FDS)

Completion Report

(for completed projects only)

Submission Deadlines:	1. *Auditor's report with unspent balance, if any: within **six** months of the approved project completion date.* 2. *Completion report: within **12** months of the approved project completion date.*

Part A: The Project and Investigator(s)

1. Project Title

Interior Subgradient Methods for Large-scale Quasi-convex Optimization and Their Applications

2. Investigator(s) and Academic Department(s) / Unit(s) Involved

Research Team	Name / Post	Unit / Department / Institution
Principal Investigator	YU Kwok-wai / Associate Professor	Department of Mathematics, Statistics and Insurance / The Hang Seng University of Hong Kong
Co-Investigator(s)	YANG Xiao Qi / Professor	Department of Applied Mathematics / The Hong Kong Polytechnic University
Co-Investigator(s)	CHOY Siu Kai / Associate Professor	Department of Mathematics, Statistics and Insurance / The Hang Seng University of Hong Kong

3. Project Duration

	Original	Revised	Date of RGC / Institution Approval *(must be quoted)*
Project Start Date	1 January 2018		
Project Completion Date	31 December 2020	30 June 2021	24 November 2020
Duration *(in month)*	36 months	42 months	24 November 2020
Deadline for Submission of Completion Report	31 December 2021	30 June 2022	24 November 2020

<u>Part B: The Final Report</u>

5. Project Objectives

5.1 Objectives as per original application

1. To propose an interior subgradient method based on the Bregman function to minimize a nondifferentiable quasi-convex function over a closed and convex set and investigate the convergence properties in terms of the function values and the Bregman distances from the iterates to the optimal solution set.

2. To introduce an incremental approach and a stochastic approach for interior subgradient methods to minimize the sum of a large number of quasi-convex component functions over a closed and convex set, respectively, and explore their convergence results and convergence rates.

3. To formulate the portfolio selection problem and the multiple Cobb-Douglas production functions efficiency problem as a quasi-concave maximization problem and a sum-maximization problem of many quasi-concave functions, respectively, and apply the proposed algorithms to solve them.

5.2 Revised objectives

Date of approval from the RGC:	NA
Reasons for the change:	NA

1.

2.

3. ..

5.3 Realisation of the objectives
(Maximum 1 page; please state how and to what extent the project objectives have been achieved; give reasons for under-achievements and outline attempts to overcome problems, if any)

Three project objectives have been addressed.

The first objective is to propose an interior subgradient method based on the Bregman function to minimize a nondifferentiable quasi-convex function over a closed and convex set. The main benefit of the interior subgradient method is that it forces the produced sequence to stay within the interior of the constraint set, and thus automatically to eliminate the Euclidean projection. To achieve this objective, we have explored different types of distance measures including Bregman distance which could be use in an interior subgradient method to solve a quasi-convex optimization problem. In addition, we have investigated the convergence properties under the Hölder condition of order p and obtained the convergence rate results under the assumptions of the Hölder condition and/or the weak sharp minima of Hölderian order.

Furthermore, we introduced an incremental approach and a stochastic approach for interior subgradient methods to minimize the sum of a large number of quasi-convex component functions over a closed and convex set, respectively (objective 2). To achieve this objective, we have investigated various approaches for solving large-scale quasi-convex optimization problems and explored the convergence properties of these approaches. We have explored the convergence results and convergence rate of the incremental approach and the stochastic approach, respectively.

Finally, we have applied the proposed algorithms to solve the portfolio selection problem and the multiple Cobb-Douglas productions efficiency problem (objective 3). To achieve this objective, we have considered the portfolio selection problem and used the proposed method to solve it. Also, we have formulated the multiple Cobb-Douglas productions efficiency problem as a sum-maximization problem of many quasi-concave functions and applied the incremental approach and stochastic approach to solve the problem. We have obtained some numerical results of the multiple Cobb-Douglas productions efficiency problem.

5.4 Summary of objectives addressed to date

Objectives *(as per 5.1/5.2 above)*	**Addressed** *(please tick)*	**Percentage Achieved** *(please estimate)*
1. To propose an interior subgradient method based on the Bregman function to minimize a nondifferentiable quasi-convex function over a closed and convex set and investigate the convergence properties in terms of the function values and the Bregman distances from the iterates to the optimal solution set.	✓	100%
2. To introduce an incremental approach and a stochastic approach for interior subgradient methods to minimize the sum of a large number of quasi-convex component functions over a closed and convex set, respectively, and explore their convergence results and convergence rates.	✓	100%
3. To formulate the portfolio selection problem and the multiple Cobb-Douglas production functions efficiency problem as a quasi-concave maximization problem and a sum-maximization problem of many quasi-concave functions, respectively, and apply the proposed algorithms to solve them.	✓	100%

6. Research Outcome

6.1 Major findings and research outcome
(Maximum 1 page; please make reference to Part C where necessary)

In this project, we investigate large-scale quasi-convex optimization problems. We establish theories and develop algorithms based on interior subgradient methods for large-scale quasi-convex optimization problems. Our proposed incremental and stochastic approaches based on interior subgradient methods are applied to solve sum-minimization problem of quasi-convex functions. One of our major findings is that the proposed methods are applied to solve the multiple Cobb-Douglas productions efficiency problem and the numerical results show that the proposed methods are efficient for solving large-scale sum of ratios problems. Moreover, we explore the convergence results including the iteration complexity and the convergence rates of the subgradient methods under the assumptions of the Hölder condition and/or the Hölder-type error bound property. The iteration complexity results validate the benefit of the stochastic control that it enjoys both advantages of low computational cost requirement and low (worse-case) iteration complexity.

6.2 Potential for further development of the research and the proposed course of action *(Maximum half a page)*

This project addresses the large-scale quasi-convex optimization problems. The portfolio selection problem and the multiple Cobb-Douglas productions efficiency problem are considered. And the proposed incremental and stochastic approaches based on interior subgradient methods are applied to solve these problems efficiently. We have established theories and developed algorithms based on interior subgradient methods for large-scale quasi-convex optimization problems. These theories and methods are useful and extendable to various industry problems in various fields such as economics, engineering, management science and various applied sciences.

7. Layman's Summary

(Describe in layman's language the nature, significance and value of the research project, in no more than 200 words)

Quasi-convex optimization is fundamental to the modelling of many problems in application fields such as economics, finance and industrial organization. The sum of ratios problem has a variety of important applications such as multi-stage stochastic shipping, government contracting, and bond portfolio optimization problems. Exploiting the structure, it can be formulated as a minimization problem of a sum of quasi-convex component functions over its constraints. However, the development of numerical algorithms for quasi-convex optimization, particularly for large-scale problems, is still in its infancy. There is a great demand for novel subgradient methods to solve a much broader class of large-scale quasi-convex optimization problems.

In this project, we have investigated a class of novel interior subgradient methods based on the Bregman function to minimize a single quasi-convex function or a sum of quasi-convex component functions over a closed and convex set. The project's long-term impact is in providing a deeper theoretical understanding of numerical methods to solve large-scale quasi-convex optimization problems and their applications in operations research, economics and finance.

Part C: Research Output

8. Peer-Reviewed Journal Publication(s) Arising Directly From This Research Project

(Please attach a copy of the publication and/or the letter of acceptance if not yet submitted in the previous progress report(s). All listed publications must acknowledge RGC's funding support by quoting the specific grant reference.)

The Latest Status of Publications									
Year of Publication	**Year of Acceptance** *(For paper accepted but not yet published)*	**Under Review**	**Under Preparation** *(optional)*	**Author(s)** *(denote the correspond-ing author with an asterisk*)*	**Title and Journal / Book** *(with the volume, pages and other necessary publishing details specified)*	**Submitted to RGC** *(indicate the year ending of the relevant progress report)*	**Attached to this Report** *(Yes or No)*	**Acknowledged the Support of RGC** *(Yes or No)*	**Accessible from the Institutional Repository** *(Yes or No)*
2019				Yaohua Hu*, Carisa Kwok Wai Yu, and Xiaoqi Yang	Incremental quasi-subgradient methods for minimizing the sum of quasi-convex functions, *Journal of Global Optimization*, 75, pp. 1003-1028. https://doi.org/10.1007/s10898-019-00818-6		**Yes**	**Yes (Annex I)**	**Yes (https://researchdb hsu.edu hk/view/publication/201900253)**
2020				Yaohua Hu, Jiawen Li, and Carisa Kwok Wai Yu*	Convergence rates of subgradient methods for quasi-convex optimization problems, *Computational Optimization and Applications*, 77, pp. 183-212. https://doi.org/10.1007/s10589-020-00194-y		**Yes**	**Yes (Annex II)**	**Yes (https://researchdb hsu.edu hk/view/publication/202000258)**
2021				Xiaotian Li, Linju Cai, Jingchao Li, Carisa Kwok Wai Yu, and Yaohua Hu*	A Survey of clustering methods via optimization methodology, *Journal of Applied and Numerical Optimization*, 3(1), pp. 151-174. doi: 10.23952/jano.3.2021.1.09		**Yes**	**Yes (Annex III)**	**Yes (https://researchdb hsu.edu hk/view/publication/202100229)**
2021				Chenchen Zu*, Xiaoqi Yang, and Carisa Kwok Wai Yu	Sparse minimax portfolio and Sharpe ratio models, *Journal of Industrial and Management Optimization*, doi: 10.3934/jimo.2021111.		**Yes**	**Yes (Annex IV)**	**Yes (https://researchdb hsu.edu hk/view/publication/202100230)**

2022				Kaiwen Meng, Hongyu Yang, Xiaoqi Yang*, and Carisa Kwok Wai Yu	Portfolio optimization under a minimax rule revisited, *Optimization,* 71(4), pp. 877-905. https://doi.org/10.1080/02331934.2021.1928665		**Yes**	**Yes (Annex V)**	**Yes (https://researchdb hsu.edu hk/view/publication/202100231)**
2022				Yaohua Hu, Gongnong Li, Carisa Kwok Wai Yu*, and Tsz Leung Yip	Quasi-convex feasibility problems: Subgradient methods and convergence rates, *European Journal of Operational Research,* 298(1), pp. 45-58. https://doi.org/10.1016/j.ejor.2021.09.029		**Yes**	**Yes (Annex VI)**	**Yes (https://researchdb hsu.edu hk/view/publication/202200052)**

9. Recognized International Conference(s) In Which Paper(s) Related To This Research Project Was / Were Delivered

(Please attach a copy of each conference abstract)

Month / Year / Place	**Title**	**Conference Name**	**Submitted to RGC** *(indicate the year ending of the relevant progress report)*	**Attached to this Report** *(Yes or No)*	**Acknowledged the Support of RGC** *(Yes or No)*	**Accessible from the Institutional Repository** *(Yes or No)*
NA	NA	NA	NA	NA	NA	NA

10. Whether Research Experience And New Knowledge Has Been Transferred / Has Contributed To Teaching And Learning

(Please elaborate)

NA

11. Student(s) Trained

(Please attach a copy of the title page of the thesis)

Name	**Degree Registered for**	**Date of Registration**	**Date of Thesis Submission / Graduation**
NA	NA	NA	NA

12. Other Impact

(e.g. award of patents or prizes, collaboration with other research institutions, technology transfer, teaching enhancement, etc.)

NA

13. Statistics on Research Outputs

	Peer-reviewed Journal Publications	Conference Papers	Scholarly Books, Monographs and Chapters	Patents Awarded	Other Research Outputs (please specify)	
					Type	No.
No. of outputs arising directly from this research project	6	NA	NA	NA	NA	NA

14. Public Access Of Completion Report

(Please specify the information, if any, that cannot be provided for public access and give the reasons.)

Information that Cannot Be Provided for Public Access	Reasons
NA	NA