RGC Ref. No.:
UGC/FDS14/P02/16
(please insert ref. above)

RESEARCH GRANTS COUNCIL
COMPETITIVE RESEARCH FUNDING SCHEMES FOR
THE LOCAL SELF-FINANCING DEGREE SECTOR

FACULTY DEVELOPMENT SCHEME (FDS)

Completion Report
(for completed projects only)

Submission Deadlines: *1. Auditor's report with unspent balance, if any: within **six** months of the approved project completion date.*
*2. Completion report: within **12** months of the approved project completion date.*

Part A: The Project and Investigator(s)

1. Project Title

On the Uncertainty of Value-at-Risk of Individual Risk

2. Investigator(s) and Academic Department(s) / Unit(s) Involved

Research Team	Name / Post	Unit / Department / Institution
Principal Investigator	Yuen Fei Lung / Associate Professor	Department of Mathematics, Statistics and Insurance / The Hang Seng University of Hong Kong
Co-Investigator(s)		
Others		

3. Project Duration

	Original	Revised	Date of RGC / Institution Approval *(must be quoted)*
Project Start Date	1 Jan 2017	1 Jan 2017	
Project Completion Date	31 Dec 2019	30 Jun 2020	2 Oct 2019
Duration *(in month)*	36	42	2 Oct 2019
Deadline for Submission of Completion Report	31 Dec 2020	30 Jun 2021	2 Oct 2019

Part B: The Final Report

5. Project Objectives

5.1 Objectives as per original application

1. *To develop a mathematically tractable uncertainty model for Value-at-Risk (VaR) and introduce a more robust way to measure risk*
2. *To identify the worst scenario measure and investigate the mathematical and financial properties of the corresponding VaR*
3. *To explore other potential applications of the model in financial economic, such as risk management, loss modelling, extreme value theory*

5.2 Revised objectives

N.A.

Date of approval from the RGC: _____

Reasons for the change: _____

_____

1.

2.

3. ..

5.3 Realisation of the objectives
(Maximum 1 page; please state how and to what extent the project objectives have been achieved; give reasons for under-achievements and outline attempts to overcome problems, if any)

All the three objectives of the project have been achieved. An uncertainty model for Value-at-Risk (VaR) has been investigated and developed so that there is a more robust way to measure risk. Our uncertainty model considers the worst scenario constructed by a f-divergence constraint. One of the key difficulties is that the VaR maximization problem is non-convex. It is transformed to an associated convex optimization problem and solved. The issues related to discrete random variables have also been studied in detail and some mathematical results have been obtained.

A concept of "robust VaR" has been introduced in the research which is defined using the VaR under the worst scenario measure. In the project, the mathematical properties and characteristics of the robust VaR, and the associated worst scenario measure, have been investigated and obtained. For both discrete and continuous random variables, the robust VaR is found to be the VaR with an increase in the confidence level where the increase is independent of the loss distribution. The proposed robust VaR demonstrates some very nice mathematical properties and preserves some intuitive financial results of simple VaR, such as asymptotic additivity under tail comonotonicity and subadditive under elliptically distributed dependence structure.

In addition, further investigation on the applications of VaR has been carried out in various disciplines of financial economic. Optimal reinsurance problems have been studied under different settings in which distortion risk measure minimization is used as the objective of the insurance companies. The additivity of VaR for risks with extreme value distributions has also been studied. Other risk management problems are also investigated, for example, the classical ruin model has been revisited. Some results have been obtained and published.

5.4 Summary of objectives addressed to date

Objectives *(as per 5.1/5.2 above)*	**Addressed** *(please tick)*	**Percentage Achieved** *(please estimate)*
1. To develop a mathematically tractable uncertainty model for Value-at-Risk (VaR) and introduce a more robust way to measure risk	✓	100%
2. To identify the worst scenario measure and investigate the mathematical and financial properties of the corresponding VaR	✓	100%
3. To explore other potential applications of the model in financial economic, such as risk management, loss modelling, extreme value theory	✓	100%

6. Research Outcome

6.1 Major findings and research outcome
(Maximum 1 page; please make reference to Part C where necessary)

In Cheung and Yuen (2020), the concept of "robust VaR" has been introduced, and the associated worst scenario of individual risk has been investigated and obtained in the project. We have found that both the continuous and discrete random variables have the same distortion on the confidence level for the robust VaR. The distortion is shown to be independent of the loss distribution, but they have different forms of the worst scenario measure. It is caused by the jump in distribution function and the mismatch with the confidence level of VaR. Three forms of Radon-Nikodym derivatives are introduced for the discrete variables in order to construct different forms of worst scenario measures for different levels of uncertainty. Some properties of the robust VaR and the worst scenario have also been demonstrated, such as comonotonic additivity.

In Cheung, Yam, and Yuen (2019), the optimal reinsurance policy provision in a monopolistic reinsurance market with adverse selection is studied. Two groups of insurers, high risk and low risk, are assumed to be a VaR minimizer, while the single reinsurer who has partial information about the risk characteristics of the insurers is maximizing its expected profit. We have found that the reinsurer has to sacrifice part of its expected profit or even give up the low-risk market to reveal the identity of the insurers. Although stop-loss contract has long been used to reduce moral hazard in asymmetric information problem, we have shown that quota-share ratio is also essential due to its stronger effect on the high-risk group and so it can help reveal the insurers' private information. In Cheung, Yam, Yuen, and Zhang (2020), the problem is extended to concave distortion risk measures for stop-loss contract. The loss of the high-risk group is assumed to stochastically dominate that of the low-risk group. The optimal reinsurance contract is obtained. We have shown that the design of the optimal contracts mainly depends on the market composition and the difference between the risk groups.

In Cheung, Ling, Tang, Yam, and Yuen (2019), we have proved the asymptotic additivity of VaR for upper tail comonotonic risks such that each marginal risk is belonging to one of the three maximum domains of attraction or having a finite endpoint. The risks do not require to be identically distributed and the results have also been extended to conditional tail expectation and tail VaR.

In Yuen, Lee, and Fung (2020), we have revisited the compound Poisson risk model with a cyclic approach and provided an intuitive way to understand some classic results in ruin theory. With the cyclically interchangeable property of claims and the weak law of large numbers, we have obtained some results on the ruin probability and distribution of deficit by considering a random starting point.

6.2 Potential for further development of the research and the proposed course of action *(Maximum half a page)*

With the result of the famous Sklar's theorem, multivariate distribution can be expressed in terms of univariate marginal distributions and a copula which describes the dependence structure among the variables. The robust VaR and the associated worst scenario of individual risk have been studied and obtained in the project. The next step is to identify the worst dependence structure for an aggregated risk. In particular, comonotonicity is usually considered to be an example of the worst dependence structure for risk quantified by coherent risk measures. In general, the form of the risk maximizing worst scenario is still unknown while important and it needs further investigation. In addition, the three maximum domains of attraction (MDAs), especially the Fréchet MDA, are commonly used to study asymptotic properties of the aggregated risk, and those component risks are usually assumed to be tail equivalent. We have released this constraint on the marginal distribution under tail comonotonicity. A potential research direction is to investigate if the tail equivalent condition can be removed under other circumstances.

7. Layman's Summary

(Describe in layman's language the nature, significance and value of the research project, in no more than 200 words)

Risk measure is a way to quantify and summarize the risk of a financial position into a single numerical value which is important for risk management, risk control, premium and reserve capital calculations. In assessing the risk measure, we usually make an assumption on the loss distribution using probability models, or we can use the empirical distributions to calculate the risk measure and predict the level of risk we are bearing. However, the models and empirical distributions are just an estimation and we do not know the true distribution of the risk. It is known as the problem of uncertainty. In this project, we have developed an uncertainty model for a commonly used risk measure, VaR, and investigated its properties. The project has also investigated various related risk management problems in actuarial science and insurance. The obtained results can also be applied in other disciplines and they have brought us a better understanding on uncertainty and risk measure.

Part C: Research Output

8. Peer-Reviewed Journal Publication(s) Arising Directly From This Research Project

(Please attach a copy of the publication and/or the letter of acceptance if not yet submitted in the previous progress report(s). All listed publications must acknowledge RGC's funding support by quoting the specific grant reference.)

The Latest Status of Publications				Author(s) *(denote the corresponding author with an asterisk*)*	Title and Journal / Book *(with the volume, pages and other necessary publishing details specified)*	Submitted to RGC *(indicate the year ending of the relevant progress report)*	Attached to this Report *(Yes or No)*	Acknowledged the Support of RGC *(Yes or No)*	Accessible from the Institutional Repository *(Yes or No)*
Year of Publication	Year of Acceptance *(For paper accepted but not yet published)*	Under Review	Under Preparation *(optional)*						
2019				K. C. Cheung, S. C. P. Yam & F. L. Yuen*	Reinsurance contract design with adverse selection / *Scandinavian Actuarial Journal*, 9, 784-798.		Yes (Annex I)	Yes	Yes
2019				Ka Chun Cheung, Hok Kan Ling, Qihe Tang, Sheung Chi Phillip Yam & Fei Lung Yuen*	On additivity of tail comonotonic risks / *Scandinavian Actuarial Journal*, 10, 837-866.		Yes (Annex II)	Yes	Yes
2020				K.C. Cheung, F.L. Yuen*	On the uncertainty of VaR of individual risk / *Journal of Computational and Applied Mathematics*, Vol. 367, 112468. https://doi.org/10.1016/j.cam.2019.112468.		Yes (Annex III)	Yes	Yes
2020				Fei Lung Yuen*, Wing Yan Lee, Derrick W.H. Fung	A cyclic approach on classical ruin model / *Insurance: Mathematics and*		Yes (Annex IV)	Yes	Yes

					***Economics*, 91, 104-110.**				
2020				**Ka Chun Cheung, Sheung Chi Phillip Yam, Fei Lung Yuen, Yiying Zhang***	**Concave distortion risk minimizing reinsurance design under adverse selection / *Insurance: Mathematics and Economics*, 91, 155-165.**		**Yes (Annex V)**	**Yes**	**Yes**

9. Recognized International Conference(s) In Which Paper(s) Related To This Research Project Was / Were Delivered
(Please attach a copy of each conference abstract)

Month / Year / Place	**Title**	**Conference Name**	**Submitted to RGC** *(indicate the year ending of the relevant progress report)*	**Attached to this Report** *(Yes or No)*	**Acknowledged the Support of RGC** *(Yes or No)*	**Accessible from the Institutional Repository** *(Yes or No)*
July/ 2017/ Vienna	On the Uncertainty of Individual Risk	21st International Congress on Insurance: Mathematics and Economics	2018	Yes (Annex VI)	No#	Yes

The conference organizer did not provide space for acknowledgement.

10. Whether Research Experience And New Knowledge Has Been Transferred / Has Contributed To Teaching And Learning
(Please elaborate)

During the investigation on the robust VaR, the key concepts of risk management and the latest development on the industrial practices were reviewed and investigated, such as the standard models using in Solvency II and the modifications in Basel Accords. These information and experience are useful for teaching and have been delivered to students in teaching. Some knowledge developed in the research problems and results can also be shared

with the students and widen their horizons in this area, and they can access the latest development of risk management and financial mathematics.

11. Student(s) Trained

(Please attach a copy of the title page of the thesis)

Name	Degree Registered for	Date of Registration	Date of Thesis Submission / Graduation
N.A.			

12. Other Impact

(e.g. award of patents or prizes, collaboration with other research institutions, technology transfer, teaching enhancement, etc.)

N.A.

13. Statistics on Research Outputs

	Peer-reviewed Journal Publications	Conference Papers	Scholarly Books, Monographs and Chapters	Patents Awarded	Other Research Outputs (please specify)	
No. of outputs arising directly from this research project	5	1			Type	No.

14. Public Access Of Completion Report

(Please specify the information, if any, that cannot be provided for public access and give the reasons.)

Information that Cannot Be Provided for Public Access	Reasons
N.A.	