RGC Ref. No.:
UGC/FDS14/P02/15
(please insert ref. above)

RESEARCH GRANTS COUNCIL
COMPETITIVE RESEARCH FUNDING SCHEMES FOR THE LOCAL SELF-FINANCING DEGREE SECTOR

FACULTY DEVELOPMENT SCHEME (FDS)

Completion Report
(for completed projects only)

Submission Deadlines:	*1. Auditor's report with unspent balance, if any: within* ***six*** *months of the approved project completion date.* *2. Completion report: within* ***12*** *months of the approved project completion date.*

Part A: The Project and Investigator(s)

1. Project Title

Sparse Optimization Models with Application to Portfolio Management

2. Investigator(s) and Academic Department(s) / Unit(s) Involved

Research Team	Name / Post	Unit / Department / Institution
Principal Investigator	YU Kwok-wai / Associate Professor	Department of Mathematics, Statistics and Insurance / The Hang Seng University of Hong Kong
Co-Investigator(s)	YANG Xiao Qi / Professor	Department of Applied Mathematics / The Hong Kong Polytechnic University
Others		

3. Project Duration

	Original	Revised	Date of RGC / Institution Approval *(must be quoted)*
Project Start Date	1 January 2016		
Project Completion Date	31 December 2018	30 June 2019	22 October 2018
Duration *(in month)*	36 months	42 months	22 October 2018
Deadline for Submission of Completion Report	31 December 2019	30 June 2020	22 October 2018

Part B: The Final Report

5. Project Objectives

5.1 Objectives as per original application

1. Study the characteristics of the Pareto optimal solution set of sparse tri-objective quadratic programming problems (STQPPs) and their first and second order optimality conditions, and devise effective computational algorithms and methods for solving STQPPs;

2. Investigate sparse bi-objective CVaR programming problems (SBCPPs) and consider multi-objective convex programming method and piecewise linear approach for solving the SBCPPs.

(A) For the multi-objective convex programming method

- Propose a novel subgradient method to solve the SBCPPs and devise effective algorithms.
- Conduct extensive numerical experiments to demonstrate the high efficiency of our proposed algorithm on solving bi-objective convex programming.

(B) In the piecewise linear approach

- Apply the multi-objective simplex method and Benson's outer approximation algorithm to devise an effective algorithm to obtain the set of all Pareto optimal solutions.
- Apply the proposed algorithms to obtain optimal sparse portfolio strategies and use the real data from the Hong Kong Stock Exchange for empirical analysis.

5.2 Revised objectives

Date of approval from the RGC: N/A

Reasons for the change:

1.

2.

3. ....

5.3 Realisation of the objectives
(Maximum 1 page; please state how and to what extent the project objectives have been achieved; give reasons for under-achievements and outline attempts to overcome problems, if any)

Two project objectives have been addressed.

The first objective is to study the characteristics of the Pareto optimal solution set of sparse tri-objective quadratic programming problems (STQPPs). Since the l_1 regularization problem has been accepted as one of the most useful methodologies for sparse optimization, we study the STQPPs with the l_1 regularization. To achieve this objective, we have explored various iterative methods (such as scalarization methods and descent methods) for solving multi-objective optimization problems and compared the efficiency of numerical algorithms. We consider a solution scheme that can capture the whole nondominated set of STQPP. In the theoretical framework, we have investigated the optimality conditions and the properties of the Pareto optimal solution set of the STQPPs.

Furthermore, we investigate sparse bi-objective CVaR programming problems (SBCPPs), the characteristics of its Pareto optimal solution set, and hence find the optimality conditions of the SBCPPs (objective 2). We consider the multi-objective convex programming method and piecewise linear approach for solving the SBCPPs. To achieve this objective, we have studied various subgradient methods for solving convex/quasiconvex optimization problems efficiently and explored convergence properties of the subgradient methods. We apply the alternative projection strategy in our algorithm and consider different step-size rules (such as constant step-size rule and diminishing step-size rule) for the subgradient methods. For the piecewise linear approach, we devise an efficient algorithm to find the Pareto optimal set for the exact solution and analyze its effectiveness and properties.

5.4 Summary of objectives addressed to date

Objectives *(as per 5.1/5.2 above)*	**Addressed** *(please tick)*	**Percentage Achieved** *(please estimate)*
1. Study the characteristics of the Pareto optimal solution set of sparse tri-objective quadratic programming problems (STQPPs) and their first and second order optimality conditions, and devise effective computational algorithms and methods for solving STQPPs;	✓	100%
2. Investigate sparse bi-objective CVaR programming problems (SBCPPs) and consider multi-objective convex programming method and piecewise linear approach for solving the SBCPPs.	✓	100%

6. Research Outcome

6.1 Major findings and research outcome
(Maximum 1 page; please make reference to Part C where necessary)

In this project, we establish theory on sparse multi-objective models and apply it to develop a portfolio strategy with better sparsity. In the study of multi-objective optimization problems, we consider an extended Newton method for approaching a Pareto optimum and establish quadratic convergence criteria. One of our findings is that the convergence theorems significantly improve the corresponding ones in some previous studies. Moreover, we propose an iterative positive thresholding algorithm for solving the non-negative sparse optimization problem. The significant advantage of the algorithm is that it is very simple and of low computational cost. With the concepts and theories of multi-objective and sparse optimization problems, we have studied the characteristics of the Pareto optimal solution set of sparse tri-objective quadratic programming problems (STQPPs) and sparse bi-objective CVaR programming problems (SBCPPs). For solving the STQPPs, we use the Clarke subgradient for analysis and devise an effective computational algorithm. For the SBCPPs, we apply the alternative projection strategy and piecewise linear approach to solve the problems efficiently.

6.2 Potential for further development of the research and the proposed course of action *(Maximum half a page)*

This project addresses the sparse multi-objective optimization problems. We have formulated a portfolio problem into a sparse tri-objective quadratic programming problem and a sparse bi-objective programming problem, respectively. These problems can be solved efficiently by our proposed methods. we also establish solution strategies and theories for these types of problems. Our methodologies and theories can be further developed to solve problems in other areas such as economy, engineering, network design and operation, supply chain management, and environmental analysis.

7. Layman's Summary

(Describe in layman's language the nature, significance and value of the research project, in no more than 200 words)

Sparse optimization has become a very popular research topic in many disciplines of applied science and gained successful applications in a wide range of fields, which aims to find a sparse approximate solution of an underdetermined linear system from the underlying data. The concept of sparse optimization is of great practical importance. One of the attractive applications is the construction of sparse portfolios. It is worth noting that most problems in the real world are concerned with more than one objective. Multi-objective optimization problem has been extensively applied in various areas. Therefore, there is a great demand to establish an in-depth theory of the optimality of sparse multi-objective optimization problems.

In this project, we have established theories on sparse multi-objective models and apply the concepts and theories to develop a portfolio strategy with better sparsity. Through the construction of an optimal portfolio, risk is identified and assessed. We provide reliable methodologies for assessing risk and make a significant contribution to risk management. The concepts and theories can be extendable to many decision-making problems.

Part C: Research Output

8. Peer-Reviewed Journal Publication(s) Arising Directly From This Research Project

(Please attach a copy of the publication and/or the letter of acceptance if not yet submitted in the previous progress report(s). All listed publications must acknowledge RGC's funding support by quoting the specific grant reference.)

The Latest Status of Publications									
Year of Publication	**Year of Acceptance** *(For paper accepted but not yet published)*	**Under Review**	**Under Preparation** *(optional)*	**Author(s)** *(denote the correspond-ing author with an asterisk*)*	**Title and Journal / Book** *(with the volume, pages and other necessary publishing details specified)*	**Submitted to RGC** *(indicate the year ending of the relevant progress report)*	**Attached to this Report** *(Yes or No)*	**Acknowledged the Support of RGC** *(Yes or No)*	**Accessible from the Institutional Repository** *(Yes or No)*
2019				Jinhua WANG, Yaohua HU*, Carisa Kwok Wai YU, Chong LI and Xiaoqi YANG	Extended Newton Methods for Multiobjective Optimization: Majorizing Function Technique and Convergence Analysis, *Siam Journal on Optimization*, 29(3), pp. 2388-2421.		**Yes**	**Yes**	**Yes**
2019				Jinhua WANG, Yaohua HU*, Carisa Kwok Wai YU and Xiaojun ZHUANG	A Family of Projection Gradient Methods for Solving the Multiple-Sets Split Feasibility Problem, *Journal of Optimization Theory and Applications*, 183, pp. 520-534.		**Yes**	**Yes**	**Yes**
2019				Jifeng BAO, Carisa Kwok Wai YU, Jinhua WANG*, Yaohua. HU, and Jen-Chih YAO	Modified inexact Levenberg–Marquardt methods for solving nonlinear least squares problems, *Computational Optimization and Applications*, 74, pp. 547-582.		**Yes**	**Yes**	**Yes**
2018				Carisa Kwok Wai YU, Yaohua HU, Xiaoqi YANG*, and Siu Kai CHOY	Abstract Convergence Theorem for Quasi-convex Optimization Problems with Applications, *Optimization*, 68(7), pp. 1289-1304.		**Yes**	**Yes**	**Yes**

2018				Lufang ZHANG, Yaohua HU*, Carisa Kwok Wai YU, and Jinhua WANG	Iterative Positive Thresholding Algorithm for Non-negative Sparse Optimization, *Optimization*, 67(9), pp. 1345-1363.		**Yes**	**Yes**	**Yes**
2018				Yan ZHANG, Yaohua HU*, Carisa Kwok Wai YU, and Jinhua WANG	Cubic Convergence of Newton-Steffensen's Method for Operators with Lipschitz Continuous Derivative, *Journal of Nonlinear and Convex Analysis*, 19(3), pp. 433-460.		**Yes**	**Yes**	**Yes**
2018				Yan ZHANG, Carisa Kwok Wai YU, Ji-feng BAO*, and Jinhua WANG	On Quadratical Convergence of Inexact Levenberg-Marquardt Methods under Local Error Bound Condition, *Journal of Nonlinear and Convex Analysis*, 19(1), pp.123-146.		**Yes**	**Yes**	**Yes**
2017				Yaohua HU*, Xiaoqi YANG and Carisa Kwok Wai YU	Subgradient Methods for Saddle Point Problems of Quasiconvex Optimization, *Pure and Applied Functional Analysis*, 2(1), pp. 83-97.	**2017**	**Yes**	**Yes**	**Yes**
2016				Yaohua HU*, Carisa Kwok Wai YU, Chong LI and Xiaoqi YANG	Conditional Subgradient Methods for Constrained Quasi-convex Optimization Problems, *Journal of Nonlinear and Convex Analysis*, 17(10), pp. 2143-2158.	**2017**	**Yes**	**Yes**	**Yes**

9. Recognized International Conference(s) In Which Paper(s) Related To This Research Project Was / Were Delivered
(Please attach a copy of each conference abstract)

Month / Year / Place	**Title**	**Conference Name**	**Submitted to RGC** *(indicate the year ending of the relevant progress report)*	**Attached to this Report** *(Yes or No)*	**Acknowledged the Support of RGC** *(Yes or No)*	**Accessible from the Institutional Repository** *(Yes or No)*
06/ 2018/ Florence, Italy	Time-frequency Analytical Tool for Stock Market Analysis	International Journal of Arts & Sciences' (IJAS) International Conference	NA	Yes	Yes	Yes

10. Whether Research Experience And New Knowledge Has Been Transferred / Has Contributed To Teaching And Learning
(Please elaborate)

NA

11. Student(s) Trained
(Please attach a copy of the title page of the thesis)

Name	**Degree Registered for**	**Date of Registration**	**Date of Thesis Submission / Graduation**
NA	NA	NA	NA

12. Other Impact
(e.g. award of patents or prizes, collaboration with other research institutions, technology transfer, teaching enhancement, etc.)

NA

13. Statistics on Research Outputs

	Peer-reviewed Journal Publications	Conference Papers	Scholarly Books, Monographs and Chapters	Patents Awarded	Other Research Outputs (please specify)	
					Type	No.
No. of outputs arising directly from this research project	9	1				

14. Public Access Of Completion Report

(Please specify the information, if any, that cannot be provided for public access and give the reasons.)

Information that Cannot Be Provided for Public Access	Reasons
N/A	