RGC Ref. No.:
UGC/FDS14/P01/20
(please insert ref. above)

RESEARCH GRANTS COUNCIL
COMPETITIVE RESEARCH FUNDING SCHEMES FOR
THE LOCAL SELF-FINANCING DEGREE SECTOR

FACULTY DEVELOPMENT SCHEME (FDS)

Completion Report
(for completed projects only)

Submission Deadlines:
1. *Auditor's report with unspent balance, if any: within **six** months of the approved project completion date.*
2. *Completion report: within **12** months of the approved project completion date.*

Part A: The Project and Investigator(s)

1. Project Title

Inference for Multiple Change-points in Piecewise Locally Stationary Time Series

2. Investigator(s) and Academic Department(s) / Unit(s) Involved

Research Team	Name / Post	Unit / Department / Institution
Principal Investigator	NG Wai-leong / Assistant Professor	Department of Mathematics, Statistics and Insurance / The Hang Seng University of Hong Kong
Co-Investigator(s)	N/A	N/A
Others	N/A	N/A

3. Project Duration

	Original	Revised	Date of RGC / Institution Approval *(must be quoted)*
Project Start Date	01 Jan 2021	N/A	N/A
Project Completion Date	31 Dec 2023	30 Jun 2024	29 May 2023 (HSUHK)
Duration *(in month)*	36	42	29 May 2023 (HSUHK)
Deadline for Submission of Completion Report	31 Dec 2024	30 Jun 2025	29 May 2023 (HSUHK)

4.4 Please attach photo(s) of acknowledgement of RGC-funded facilities / equipment.
N/A

Part B: The Final Report

5. Project Objectives

5.1 Objectives as per original application

1. (Efficient change-points estimation) To develop a nonparametric scan statistic for fast computation and a model selection approach using minimum description length information criterion for estimation of the number and locations of change-points in piecewise locally stationary processes. To build practical and computationally efficient algorithms for the proposed method.

2. (Construction of confidence intervals) To study the asymptotic behavior of the change-point estimators and develop bootstrap procedures for construction of confidence interval for change-points.

3. (Asymptotic statistical properties) To examine the consistency of estimated number and locations of change-points and the validity of the proposed bootstrap procedures in construction of confidence intervals. By means of simulation experiments, to thoroughly examine the applications of the proposed method on various models with a wide variety of stochastic structures and illustrate the application of the proposed method to real data sets.

5.2 Revised objectives

Date of approval from the RGC: N/A

Reasons for the change: N/A

1.

2.

3. ..

5.3 Realisation of the objectives
(Maximum 1 page; please state how and to what extent the project objectives have been achieved; give reasons for under-achievements and outline attempts to overcome problems, if any)

All three project objectives have been achieved.

Objective 1: (Efficient change-points estimation) To develop a nonparametric scan statistic for fast computation and a model selection approach using minimum description length information criterion for estimation of the number and locations of change-points in piecewise locally stationary processes. To build practical and computationally efficient algorithms for the proposed method.

- We have developed a three-step procedure for multiple change-point inference in a piecewise locally stationary time series with possibly jumps and kinks in its parameter curve. In the first step, a quick detection for a set of potential change-points using the proposed nonparametric frequency-domain-based scan statistics which measure the difference between segments is used. In the second step, a model selection procedure using a minimum description length (MDL) information criterion is employed to estimate the number and locations of change-points.

Objective 2: (Construction of confidence intervals) To study the asymptotic behavior of the change-point estimators and develop bootstrap procedures for construction of confidence interval for change-points.

- For the third step of the proposed procedure, a refinement estimation of the location for each change-point is employed. The asymptotic distribution of the change-point estimator is established, and a parametric bootstrap method is developed to construct confidence interval for each estimated change-point.

Objective 3: (Asymptotic statistical properties) To examine the consistency of estimated number and locations of change-points and the validity of the proposed bootstrap procedures in construction of confidence intervals. By means of simulation experiments, to thoroughly examine the applications of the proposed method on various models with a wide variety of stochastic structures and illustrate the application of the proposed method to real data sets.

- We have established theoretical asymptotic properties of the proposed procedure including consistency of the number and locations of the change-point estimation and the asymptotic exactness of the proposed bootstrap confidence intervals. Simulation studies and real data applications to the financial time series are also conducted to illustrate the performance of the proposed method.

5.4 Summary of objectives addressed to date

Objectives *(as per 5.1/5.2 above)*	**Addressed** *(please tick)*	**Percentage Achieved** *(please estimate)*
1. (Efficient change-points estimation) To develop a nonparametric scan statistic for fast computation and a model selection approach using minimum description length information criterion for estimation of the number and locations of change-points in piecewise locally stationary processes. To build practical and computationally efficient algorithms for the proposed method.	✓	100%
2. (Construction of confidence intervals) To study the asymptotic behavior of the change-point estimators and develop bootstrap procedures for construction of confidence interval for change-points.	✓	100%
3. (Asymptotic statistical properties) To examine the consistency of estimated number and locations of change-points and the validity of the proposed bootstrap procedures in construction of confidence intervals. By means of simulation experiments, to thoroughly examine the applications of the proposed method on various models with a wide variety of stochastic structures and illustrate the application of the proposed method to real data sets.	✓	100%
4. N/A	N/A	N/A

6. Research Outcome

6.1 Major findings and research outcome
(Maximum 1 page; please make reference to Part C where necessary)

1. We have developed two bootstrap procedures, namely parametric and block bootstrap, to approximate the finite sample distribution of change-point estimators for piecewise stationary time series. The bootstrap procedures are then used to develop a generalized likelihood ratio scan method (GLRSM) for multiple changepoint inference in piecewise stationary time series, which estimates the number and locations of change-points and provides confidence intervals for each changepoint. The computational complexity of using GLRSM for multiple change-point detection is as low as O(n(logn)^3) for a series of length n. Extensive simulation studies are provided to demonstrate the effectiveness of the proposed methodology under different scenarios. Applications to financial time series are also illustrated.

2. We have developed a frequency domain bootstrap method for random fields on Z^2. Three frequency domain bootstrap schemes are proposed to bootstrap Fourier coefficients of observations. Then, inverse transformations are applied to obtain resamples in the spatial domain. As a main result, we established the invariance principle of the bootstrap samples, from which it follows that the bootstrap samples preserve the correct second-order moment structure for a large class of random fields. The frequency domain bootstrap method is simple to apply and is demonstrated to be effective in various applications including constructing confidence intervals of correlograms for linear random fields, testing for signal presence using scan statistics, and testing for spatial isotropy in Gaussian random fields. Simulation studies are conducted to illustrate the finite sample performance of the proposed method and to compare with the existing spatial block bootstrap and subsampling methods.

3. We have developed a three-step procedure to detect multiple change-points in piecewise locally stationary time series with possibly jumps and kinks in its parameter curve. The proposed method is efficient in detecting the number and locations of the change-points. By applying parametric bootstrap, we can further construct the confidence interval for the locations of the change-points with a jump. Also, by establishing the asymptotic normality of the relative change-point location estimator, we can further construct the confidence interval for the locations of the change-points with a kink, which makes our method more comprehensive in comparison with other methods in the literature. We proved that the proposed method can correctly estimate the number and locations of the change-points with probability converging to 1 when the length of the time series tends to infinity. Moreover, based on the results in the simulated studies and real data analysis, the proposed method has good performance in detecting the change-points in finite samples. Even the piecewise tvAR assumption is violated, the proposed method can still perform well in practice.

6.2 Potential for further development of the research and the proposed course of action *(Maximum half a page)*

The project can be further developed by studying the following aspects:

- Besides detecting jump and kink change-points in the parameter curves, the proposed method can potentially be extended to detecting abrupt changes in higher order derivatives in the parameter curves by using modified scan statistics.
- The use of MDL criterion in the proposed method is not restricted to piecewise tvAR models. With further treatment in the asymptotic theory on locally stationary process, the proposed method can potentially be extended to a more general setting.

- Nonparametric bootstrap and frequency-domain bootstrap methods can potentially be developed and applied to construct confidence intervals for the jump and kink change-points in piecewise locally stationary processes.

7. Layman's Summary

(Describe <u>in layman's language</u> the nature, significance and value of the research project, in no more than 200 words)

In this research project, we have developed a three-step procedure to detect multiple change-points in piecewise locally stationary time series with possibly jumps and kinks in its parameter curve. The proposed method is efficient in detecting the number and locations of the change-points. By applying parametric bootstrap method, we can construct the confidence interval for the location of each change-point, which makes our method more comprehensive in comparison with other methods in the literature.

We established that the proposed method can correctly estimate the number and locations of the change-points with probability converging to 1 when the length of the time series tends to infinity. Moreover, based on the results in the simulated studies and real data analysis, the proposed method has good performance in detecting the change-points in finite samples. If the piecewise time-varying autoregressive (tvAR) model assumption is violated, the proposed method can still perform well in practice. We believe that the use of minimum description length information criterion in the proposed method is not restricted to piecewise tvAR models. With further treatment in the asymptotic theory on locally stationary process, the proposed method can potentially be extended to a more general setting.

Part C: Research Output

8. Peer-Reviewed Journal Publication(s) Arising Directly From This Research Project

(Please attach a copy of the publication and/or the letter of acceptance if not yet submitted in the previous progress report(s). All listed publications must acknowledge RGC's funding support by quoting the specific grant reference.)

The Latest Status of Publications				Author(s) *(denote the corresponding author with an asterisk*)*	Title and Journal / Book *(with the volume, pages and other necessary publishing details specified)*	Submitted to RGC *(indicate the year ending of the relevant progress report)*	Attached to this Report *(Yes or No)*	Acknowledged the Support of RGC *(Yes or No)*	Accessible from the Institutional Repository *(Yes or No)*
Year of Publication	**Year of Acceptance** *(For paper accepted but not yet published)*	**Under Review**	**Under Preparation** *(optional)*						
2022				**NG Wai Leong*, PAN Shenyi, and YAU Chun Yip**	**Bootstrap Inference for Multiple Change-points in Time Series. Econometric Theory, 38(4), 752-792.**	**Yes (Mar 2022)**	**Yes**	**Yes (Annex I)**	**Yes https://researchdb.hsu.edu.hk/view/publication/2022200298**
2021				**NG Wai Leong*, YAU Chun Yip, and CHEN Xinyuan**	**Frequency Domain Bootstrap Methods for Random Fields. Electronic Journal of Statistics, 15(2), 6586-6632.**	**No**	**Yes**	**Yes (Annex II)**	**Yes https://researchdb.hsu.edu.hk/view/publication/2021100182**
			2025	**NG Wai Leong*, TANG Xinyi, CHEUN Mun Lau, GAO Jiacheng, YAU Chun Yip, and DETTE Holger**	**Inference for Multiple Change-points in Piecewise Locally Stationary Time Series.**	**No**	**No**	**Yes**	**No**

9. Recognized International Conference(s) In Which Paper(s) Related To This Research Project Was / Were Delivered
(Please attach a copy of each conference abstract)

Month / Year / Place	**Title**	**Conference Name**	**Submitted to RGC** *(indicate the year ending of the relevant progress report)*	**Attached to this Report** *(Yes or No)*	**Acknowledged the Support of RGC** *(Yes or No)*	**Accessible from the Institutional Repository** *(Yes or No)*
Aug / 2023 / Japan	Inference for Multiple Change-points in Piecewise Locally Stationary Time Series	The 6th International Conference on Econometrics and Statistics	No	Yes (Annex III)	Yes (in the talk)	No
Jun / 2024 / USA	Inference for Multiple Change-points in Piecewise Locally Stationary Time Series	The ICSA 2024 Applied Statistics Symposium	No	Yes (Annex IV)	Yes (in the talk)	No

10. Whether Research Experience And New Knowledge Has Been Transferred / Has Contributed To Teaching And Learning
(Please elaborate)

The key ideas and concepts of change-point analysis can be presented in undergraduate courses related to time series. Meanwhile, the concepts of bootstrap resampling can be included in courses focused on data mining and machine learning.

11. Student(s) Trained
(Please attach a copy of the title page of the thesis)

Name	**Degree Registered for**	**Date of Registration**	**Date of Thesis Submission / Graduation**
N/A	N/A	N/A	N/A

12. Other Impact

(e.g. award of patents or prizes, collaboration with other research institutions, technology transfer, teaching enhancement, etc.)

N/A

13. Statistics on Research Outputs

	Peer-reviewed Journal Publications	Conference Papers	Scholarly Books, Monographs and Chapters	Patents Awarded	Other Research Outputs (please specify)	
No. of outputs arising directly from this research project	2	2	0	0	Type	No.
					N/A	N/A

14. Public Access Of Completion Report

(Please specify the information, if any, that cannot be provided for public access and give the reasons.)

Information that Cannot Be Provided for Public Access	Reasons
N/A	N/A