

RGC Ref. No.: UGC/FDS13/E01/20 <p>(please insert ref. above)</p>
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**RESEARCH GRANTS COUNCIL  
COMPETITIVE RESEARCH FUNDING SCHEMES FOR  
THE LOCAL SELF-FINANCING DEGREE SECTOR**

**FACULTY DEVELOPMENT SCHEME (FDS)**

**Completion Report**  
(for completed projects only)

<p><b><u>Submission Deadlines:</u></b></p> <ol style="list-style-type: none"> <li>1. Auditor's report with unspent balance, if any: within <b><u>six</u></b> months of the approved project completion date.</li> <li>2. Completion report: within <b><u>12</u></b> months of the approved project completion date.</li> </ol>
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**Part A: The Project and Investigator(s)**

**1. Project Title**

Truthful Mechanism Design for Facility Location Game with Agents' Migration  
Scheme

**2. Investigator(s) and Academic Department(s) / Unit(s) Involved**

Research Team	Name / Post	Unit / Department / Institution
Principal Investigator	Jan 2021 – Jul 2022 Dr FONG Chi Kit Ken, Assistant Professor	Computer Science, Chu Hai College of Higher Education
	Aug 2022 – Present Prof Lo Wai Lun, Head, Professor	Computer Science, Faculty of Science and Engineering, Hong Kong Chu Hai College
Co-Investigator(s)	Aug 2022 – present Dr FONG Chi Kit Ken, Assistant Professor of Teaching	Division of AI, School of Data Science, Lingnan University
	Dr Vincent Chau Associate Professor	Up to Feb 2021 Shenzhen Institutes of Advanced Technology, Chinese Academy of Science. Mar 2021 – present School of Computer Science and Engineering, Southeast University, China
	Dr Hau Chan, Assistant Professor	University of Nebraska-Lincoln, Lincoln, NE, the USA.
Others		

### 3. Project Duration

	Original	Revised	Date of RGC / Institution Approval (must be quoted)
Project Start Date	1 Jan 2021	N.A	N.A
Project Completion Date	30 Jun 2023	30 Jun 2024	First extension for 6 months: Institution approved on 17 May 2023, reported to RGC on 18 May 2023; Second extension for additional 6 months: RGC approved on 5 Dec 2023
Duration ( <i>in month</i> )	30	42	
Deadline for Submission of Completion Report	30 Jun 2024	30 Jun 2025	

4.4 Please attach photo(s) of acknowledgement of RGC-funded facilities / equipment.

N.A.

## **Part B: The Final Report**

### 5. Project Objectives

#### 5.1 Objectives as per original application

1. To study the extension of capacity constraint model proposed by Aziz et al. and design truthful mechanisms for the multi-stage packing model which is a multi-stage setting of capacity constraint model proposed by Aziz et al.
2. To study the problem with k-homogeneous facilities and design strategy-proof mechanisms with good approximation ratios or prove negative results.
3. To extend the problem by introducing a new variable – tolerance rate (where the agents cannot stay for the next rounds forever). We aim to start with uniform case and then extend to heterogeneous tolerance rate when designing strategy-proof mechanisms.
4. To study the problem for the optional preference setting where agents might be in different between more than one facility. Measures adopted in the classic setting will also be studied.

#### 5.2 Revised objectives

Date of approval from the RGC: N.A.

Reasons for the change: N.A.

1.

2.

3. ....

### 5.3 Realisation of the objectives

*(Maximum 1 page; please state how and to what extent the project objectives have been achieved; give reasons for under-achievements and outline attempts to overcome problems, if any)*

#### *Realisation of Objective 1 and 3:*

We start by decomposing the problem into a multi-stage facility location problem without capacity constraints. A new variable, termed the tolerant rate, is introduced, and we classify this type of agent as a transient agent, which arrives at a specific stage and stays for several consecutive stages. Initially, we concentrate on developing optimal algorithms to minimize two objectives: the social cost objective, defined as the total distance of all agents to the facility across all stages, and the maximum cost objective, defined as the maximum distance of any agent to the facility throughout all stages. For each of these objectives, we propose a slice-wise polynomial (XP) algorithm and demonstrate that a polynomial-time algorithm exists when a natural first-come-first-served (FCFS) order for serving agents is applied. Next, we address the mechanism design problem, where the locations and arrival stages of the agents are private information. We design a group strategy-proof mechanism that achieves favorable approximation ratios for both objectives in settings with and without FCFS ordering. Finally, we extend the problem setting to include the facility's moving cost between adjacent stages under the social cost objective, which considers the total distance the facility moves. We propose slice-wise polynomial (XP) and polynomial-time algorithms, along with a group strategy-proof mechanism for both settings with and without FCFS ordering.

#### *Realisation of Objective 1 and 2:*

With regard to the capacity constraint in the multi-stage facility location problem, our initial objective is to position at most one capacity-constrained facility at each stage to serve a subset of agents arriving at different stages along a line. We begin by analyzing two different facility capacity configurations that include waiting costs: the equal capacity setting (where each stage has the same capacity) and the arbitrary capacity setting (where capacity varies across stages). For each model, we propose randomized strategy-proof mechanisms to elicit the true information of the agents and to identify facility locations that minimize both the social cost and the maximum cost. The social cost is defined as the total of the agents' costs, while the maximum cost refers to the highest cost incurred by any individual agent. Subsequently, we simplify the problem by eliminating the waiting cost and introducing a new variable, spare capacity, which allows for the capacity to exceed the total number of agents. This variable can be incorporated into both the equal and arbitrary capacity settings. In this extension, we propose both deterministic and randomized mechanisms to tackle the problem.

#### *Realisation of Objective 4:*

In relation to the optional preference model, we revisit the problem setting: each agent has a public location preference among the  $k$  facility locations and a private optional preference regarding their preferred or acceptable set of facilities from these  $k$  options. To the best of our knowledge, most existing research has focused on the two-facility location scenario within the optional preference model. We extend this work to accommodate  $k$  facilities where  $k \geq 3$ . We propose deterministic strategy-proof mechanisms to effectively address this problem.

## 5.4 Summary of objectives addressed to date

<b>Objectives</b> <i>(as per 5.1/5.2 above)</i>	<b>Addressed</b> <i>(please tick)</i>	<b>Percentage Achieved</b> <i>(please estimate)</i>
1. Multi-Stage Packing Model	✓	100%
2. Multi-Stage Packing Model with K homogeneous facilities	✓	100%
3. Multi-Stage Packing Model with Tolerance Rate	✓	100%
4. To study the problem for the optional preference setting where agents might be in different between more than one facility. Measures adopted in the classic setting will also be studied.	✓	100%

## 6. Research Outcome

### 6.1 Major findings and research outcome

*(Maximum 1 page; please make reference to Part C where necessary)*

In [1], We study various models for the one-dimensional multi-stage facility location problems with transient agents, where a transient agent arrives in some stage and stays for a number of consecutive stages. In the problems, we need to serve each agent in one of their stages by determining the location of the facility at each stage.

In the first model, we assume there is no cost for moving the facility across the stages.

- We focus on optimal algorithms to minimize both the social cost objective, defined as the total distance of all agents to the facility over all stages, and the maximum cost objective, defined as the max distance of any agent to the facility over all stages. For each objective, we give a slice-wise polynomial (XP) algorithm and show that there is a polynomial-time algorithm when a natural first-come-first-serve (FCFS) order of agent serving is enforced.
- We then consider the mechanism design problem, where the agents' locations and arrival stages are private, and design a group strategy-proof mechanism that achieves good approximation ratios for both objectives and settings with and without FCFS ordering.

In the second model, we consider the facility's moving cost between adjacent stages under the social cost objective, which accounts for the total moving distance of the facility. For this model, we design slice-wise polynomial (XP) (and polynomial time) algorithms and a group strategy-proof mechanism for settings with or without the FCFS ordering.

In [2], we study the multi-stage facility location problem with  $k$  capacity constrained facilities on the real line  $I = [0, 1]$ , where  $k$  is the maximum number of available facilities, and the real line is the most widely studied setting that abstractly represents the facility serving range (e.g., the location range of the mobile vaccine vehicle on a street, and the difficulty scale of the standardized materials). We characterize our mechanism design results for minimizing the two cost objectives by the number of agents, the number of facilities, facility capacities, the last stage with arriving agents (i.e.,  $T$ ), and the penalty coefficient (i.e.,  $d$ ).

We consider two different facility capacity settings with waiting cost (i.e.,  $d$ ).

- For the equal capacity setting (e.g., when the facility capacity cannot change over time, such as shuttle buses). we first present a randomized strategy-proof mechanism, which has a good performance when  $d$  is relatively large. We also provide another strategy-proof mechanism which performs well when  $d$  is small.
- For the arbitrary capacity setting (e.g., when the facility capacity can change over time such as a classroom in the education setting), we first provide a strategy-proof mechanism with dynamic programming to optimize the waiting cost of agents, which performs well when the waiting cost is large. Then we provide another strategy-proof mechanism with dynamic programming to optimize the distance cost of agents, which has a good performance when  $d$  is small.

## 6.2 Potential for further development of the research and the proposed course of action (Maximum half a page)

In this project, the primary goal is to examine the multiple-stage facility location model, originally proposed by Aziz et al., while incorporating capacity constraints, waiting times, and tolerance rates. We have introduced various deterministic and randomized approaches to address the challenges presented by this model. Furthermore, we have established the lower bound for certain configurations, which opens up several avenues for future development.

Most of the research in facility location models has concentrated on single-stage facility problems, leaving a gap in the exploration of multi-stage facility models. Therefore, it is clear that our next objective is to integrate more existing models into the multi-stage facility location framework. For example, we aim to incorporate the fractional model, the optional preference model, and others.

## 7. Layman's Summary

*(Describe in layman's language the nature, significance and value of the research project, in no more than 200 words)*

This research project focuses on optimizing the placement of facilities, such as warehouses or service centres, over multiple stages while considering key factors like capacity constraints, waiting times, and customer tolerance for delays. Each facility has a limited capacity to serve customers, and understanding waiting times is crucial for enhancing customer satisfaction. Additionally, we are expanding the study of customer preferences from scenarios involving two facility options to those with multiple choices (more than two). The significance of this research lies in its practical applications for businesses and service providers. By determining the best locations for facilities and effectively managing customer preferences, organizations can improve operational efficiency and reduce costs. The value of this research is multifaceted: it aims to enhance the customer experience by minimizing waiting times and ensuring convenient facility locations. This leads to increased customer loyalty and satisfaction. Furthermore, businesses can achieve greater efficiency by maximizing resource use and improving strategic planning. Overall, this project seeks to provide actionable insights and solutions that benefit both businesses and their customers across various industries.

**Part C: Research Output****8. Peer-Reviewed Journal Publication(s) Arising Directly From This Research Project**

*(Please attach a copy of the publication and/or the letter of acceptance if not yet submitted in the previous progress report(s). All listed publications must acknowledge RGC's funding support by quoting the specific grant reference.)*

The Latest Status of Publications				Author(s) (denote the corresponding author with an asterisk*)	Title and Journal / Book (with the volume, pages and other necessary publishing details specified)	Submitted to RGC (indicate the year ending of the relevant progress report)	Attached to this Report (Yes or No)	Acknowledged the Support of RGC (Yes or No)	Accessible from the Institutional Repository (Yes or No)
Year of Publication	Year of Acceptance (For paper accepted but not yet published)	Under Review	Under Preparation (optional)						
N.A.									

**9. Recognized International Conference(s) In Which Paper(s) Related To This Research Project Was / Were Delivered**

*(Please attach a copy of each conference abstract)*

Month / Year / Place	Title	Conference Name	Submitted to RGC (indicate the year ending of the relevant progress report)	Attached to this Report (Yes or No)	Acknowledged the Support of RGC (Yes or No)	Accessible from the Institutional Repository (Yes or No)
Feb 23 Washington, DC, USA	Multi-Stage Facility Location Problems with Transient Agents ( <a href="https://doi.org/10.1609/aaai.v37i5.25725">https://doi.org/10.1609/aaai.v37i5.25725</a> )	Association for the Advancement of Artificial Intelligence	No	Yes Attachment [1]	Yes	Yes
Jul 24, PolyU, Hong Kong	Randomized Strategyproof Mechanisms for Multi-Stage Facility Location Problem with Capacity Constraints ( <a href="https://doi.org/10.1007/978-981-97-7752-5_17">https://doi.org/10.1007/978-981-97-7752-5_17</a> )	International Workshop on Frontiers in Algorithmics	No	Yes Attachment [2]	Yes	Yes

**10. Whether Research Experience And New Knowledge Has Been Transferred / Has Contributed To Teaching And Learning**

*(Please elaborate)*

N.A.

**11. Student(s) Trained**

*(Please attach a copy of the title page of the thesis)*

Name	Degree Registered for	Date of Registration	Date of Thesis Submission / Graduation
	Computer Science, CityU	Sep, 2018	2022 Attachment [3]
	Computer Science, CityU	Sep, 2020	2025

**12. Other Impact**

*(e.g. award of patents or prizes, collaboration with other research institutions, technology transfer, teaching enhancement, etc.)*

This project is collaborated with the mechanism design team in the Department of Computer Science in CityU that supervised by \_\_\_\_\_, which is also Dr. Fong's PhD supervisor.

\_\_\_\_\_, was a FYP student of Prof Li, which involved in one of our research objectives, and published the paper in AAAI 23.

Another student, \_\_\_\_\_, who was a placement student that recruited under this project, has already participated in two objectives and published the paper in IJTCS-FAW 2024 and also finished another working paper which is submitted to ECAI 2025 and pending for reviews.

To further advance our research objectives, Dr. Ken FONG visited Kyushu University from June 19 to June 30, 2023, where he collaborated closely with Dr. Taiki Todo. They focused on three main research directions: extending their collaborative work, discussing multi-stage facility location problems with capacity constraints, and exploring K-facility location games with optional preferences.

Throughout the visit, they engaged in deep discussions, developed strategies for various settings, and addressed feedback from previous paper rejections.

Additionally, Dr Fong attended a postdoctoral meeting where he learned about diverse research topics, including fair event scheduling and infrastructure repair in developing countries, which align with our project's research objectives.



**13. Statistics on Research Outputs**

	<b>Peer-reviewed Journal Publications</b>	<b>Conference Papers</b>	<b>Scholarly Books, Monographs and Chapters</b>	<b>Patents Awarded</b>	<b>Other Research Outputs (please specify)</b>	
<b>No. of outputs arising directly from this research project</b>		2 delivered  1 under review Attachment [4]			Type	No.

**14. Public Access Of Completion Report**

*(Please specify the information, if any, that cannot be provided for public access and give the reasons.)*

<b>Information that Cannot Be Provided for Public Access</b>	<b>Reasons</b>
N/A	