

**PROCORE - FRANCE/HONG KONG JOINT RESEARCH SCHEME  
COMPLETION REPORT**

**Project Reference Number**

F-HK16/10T

**Project Title**

Investigation of Boundary Singularities of Some Conformally Invariant Semilinear Elliptic Problems  
共形不變半綫性方程的邊界奇性研究

**Particulars**

	Hong Kong team				French team			
Name of Project Co-ordinator (with title)	English: Professor Juncheng WEI Chinese: 魏軍城教授				Professor Frederic ROBERT			
Name of Co-Investigator (if any)	English: Chinese:							
Institution or Institutional affiliation	<input type="checkbox"/>	CityU	<input type="checkbox"/>	HKU	<input type="checkbox"/>	CEA	<input type="checkbox"/>	INRA
	<input checked="" type="checkbox"/>	CUHK	<input type="checkbox"/>	HKUST	<input type="checkbox"/>	CNRS No.	<input type="checkbox"/>	INRIA
	<input type="checkbox"/>	HKBU	<input type="checkbox"/>	LU	<input type="checkbox"/>	INFREMER	<input type="checkbox"/>	INSERM No.
	<input type="checkbox"/>	HKIED	<input type="checkbox"/>	PolyU	<input checked="" type="checkbox"/>	University of	Henri Poincare Nancy 1	
	<input type="checkbox"/>		<input type="checkbox"/>		<input type="checkbox"/>	Others:		
Other project team members (if any)								

**Funding Period**

	1 <sup>st</sup> year	2 <sup>nd</sup> year (if applicable)
Start Date	1 January 2011	1 January 2012
Completion Date	31 December 2011	31 December 2012

**Objective(s) as per original application**

1. Lin-Ni's problem for small parameters
2. Lin-Ni's problem for large parameters
3. Steklov-type problems

[Please attach relevant document(s)]

**i) Outline of proposed research and results obtained**

In this project, we proposed to study the Lin-Ni's conjecture. Results (and papers) are

1. O. Druet, F. Robert, J. Wei, The Lin-Ni's problem for mean convex domains, *Memoirs of the American Mathematical Society*, Volume 218, Number 1027, July 2012, 105pages

In this paper, we prove some refined asymptotic estimates for positive blow-up solutions to Lin-Ni problem. In particular, for dimensions  $N=3$  and  $N > 6$  we proved the Lin-Ni's Conjecture for mean convex domains and solutions with bounded energy.

2. E. Hebey and J. Wei, Schrodinger-Poisson systems in the 3-sphere, *Cal. Var. PDE* 47(2013), no.1-2, 25-54.

In this paper, we considered the extensions of Lin-Ni's problems to elliptic systems and we Studied the Schrodinger-Poisson systems with critical exponents. We proved a priori estimate and established the existence of solutions as well.

3. E. Hebey and J. Wei, Resonant states for the static Klein-Gordon-Maxwell-Proca system, *Mathematical Research Letters* 19(2012), no.4, 957-967.

In this paper, we studied the existence and stability of Klein-Gordon-Maxwell-Proca systems in general Dimensions.

4. C.S. Lin, J. Wei and D. Ye, Classification and nondegeneracy of  $SU(n+1)$  Toda system with singular sources, *Inventiones Mathematicae* 190(2012), no.1, 169-207.

In this paper, we gave a complete classification of  $SU(n+1)$  Toda systems.

5. J. Wei, B. Xu and W. Yang, On Lin-Ni's conjecture in dimensions four and six, preprint.

In this paper, we disprove Lin-Ni's conjecture for any four dimensional and six dimensional domains.

**ii) Significance of research results**

All the papers are published in top mathematical journals. The HK PI has been invited to give a 45-minute invited talk at the International Congress of Mathematicians 2014.

**iii) Research output**

1. O. Druet, F. Robert, J. Wei, The Lin-Ni's problem for mean convex domains, *Memoirs of the American Mathematical Society*, Volume 218, Number 1027, July 2012, 105pages
2. E. Hebey and J. Wei, Schrodinger-Poisson systems in the 3-sphere, *Cal. Var. PDE* 47(2013), no.1-2, 25-54.
3. E. Hebey and J. Wei, Resonant states for the static Klein-Gordon-Maxwell-Proca system, *Mathematical Research Letters* 19(2012), no.4, 953-967.
4. C.S. Lin, J. Wei and D. Ye, Classification and nondegeneracy of  $SU(n+1)$  Toda system with singular sources, *Inventiones Mathematicae* 190(2012), no.1, 169-207.
5. J. Wei, B. Xu and W. Yang, On Lin-Ni's conjecture in dimensions four and six, preprint.

**iv) Potential for or impact on further research collaboration**

This grant is very difficult and inconvenient to use.