

**PROCORE - FRANCE/HONG KONG JOINT RESEARCH SCHEME
COMPLETION REPORT**

Project Reference Number

F-HK026/12T

Project Title

Trace Formulas and Their Applications

Particulars

	Hong Kong team				French team			
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Funding Period

	1 st year	2 nd year (if applicable)
Start Date	Jan 1, 2013	Jan 1, 2014
Completion Date	Dec 31, 2013	Dec 31, 2014

Objective(s) as per original application

1. Unify two ways of counting in the analytic theory of automorphic form, and interpret this unification in terms of the relative trace formula. Our objective is then to use this unification to count Hecke coefficients of Hilbert Modular Forms.
2. Explore some geometric counting problems with the tools of Analytic Number Theory to compute error terms in some known asymptotics.

[Please attach relevant document(s)]

i) Outline of proposed research and results obtained

We introduce a weight in the average which provides a continuous path from one average to the other, showing that these two averages are particular cases of an infinite family of possible average operators. (cf [1])

We count the number of sign changes in the sequence of the Fourier coefficients of half-integral weight modular Hecke eigenforms over squarefree integers. (cf. [2])

ii) Significance of research results

The work in [1] unifies the natural averaging and the harmonic averaging so that one can see more transparently the formation of the density function. Moreover, this work extends the investigation in level aspect by Lau, Royer & Wu to the weight aspect. Furthermore, it corrects some erroneous/oversimplified calculation in a paper of Q. Sun in relevant to our study.

It is widely known that the counting of integral points on a surface given by a quadratic form in odd dimensional spaces is closely related to the modular form of half-integral weight. E.g. For a (positive-definite) diagonal form the surface is an ellipsoid, or even a sphere. Quite often, after subtracted with the expected number, the remnant/error will be encoded in the Fourier coefficient of a (half-weight) cusp form. Ultimately the oscillatory nature of the error is revealed in the sign-change problem of these Fourier coefficients. The work in [2] gives a quantitative result for the number sign-changes of the errors for the surfaces parametrized by squarefree numbers. This strengthened a joint work of Hulse, Kiral, Chan & Lim where they gave a qualitative result, i.e. no explicit estimate is given.

iii) Research output

[1] Y.-K. Lau, C. Li, M.H. Ng, E. Royer, J. Wu, "Moments of automorphic L -functions in weight aspect", in preparation.

[2] Y.-K. Lau, E. Royer, J. Wu, "Sign of Fourier coefficients of modular forms of half integral weight", submitted (18 pages).

iv) Potential for or impact on further research collaboration

We are attempting various methods to derive the number of sign-changes of the half-integral weight modular Hecke eigenforms. In particular we would try to study the mean square formula/estimates for the Fourier coefficients over squarefree numbers. No such a result is found in the literature. We shall continue our collaboration of research in this problem and explore further the applications.